CYBER-PHYSICAL SYSTEM IMPLEMENTATION INTO THE DISTRIBUTED CONTROL SYSTEM

Anna JADLOVSKÁ
Slávka JADLOVSKÁ
Dominik VOŠČEK

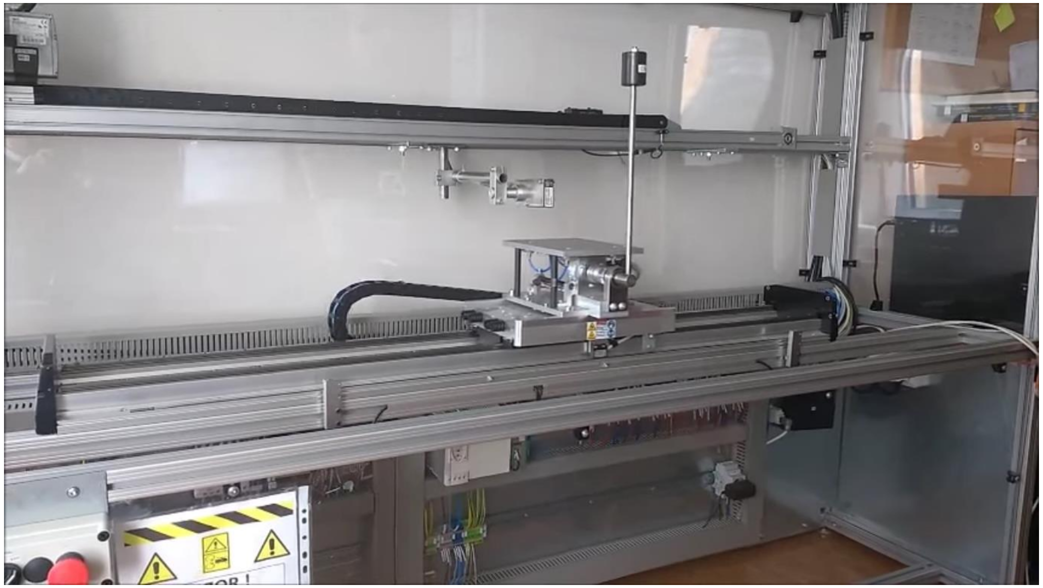
MOTIVATION – CYBER-PHYSICAL SYSTEMS

- S. Jadlovská, PhD: *Modeling and Optimal Control of Nonlinear Underactuated Dynamical Systems* (PhD thesis supervised by prof. J. Sarnovský, defended in 2015)
- D. Voščžek: *Hybrid Models of Cyber-Physical Systems and Their Application into Distributed Control Systems* (PhD student supervised by doc. A. Jadlovská, 2015-)

- **Cyber-Physical Systems**

  integrate physical processes with computations

  provide interactions among computing, control, physical & network systems



- **Multipurpose Workplace for Nondestructive Diagnostics with Linear Synchronous Motor – Single Inverted Pendulum Laboratory Model**

OVERVIEW OF THE PRESENTED RESULTS

Inverted Pendula Modeling and Control Block Library

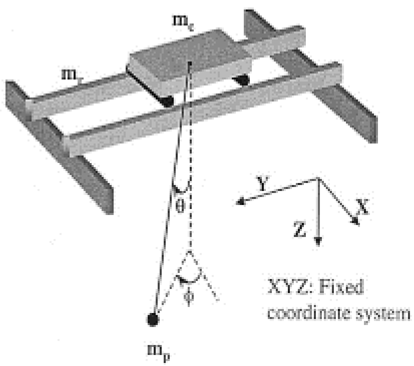
- Modeling of Underactuated Mechanical Systems Using Classical Mechanics
- Optimal Control of Underactuated Dynamical Systems
- Hybrid Control of Underactuated Systems

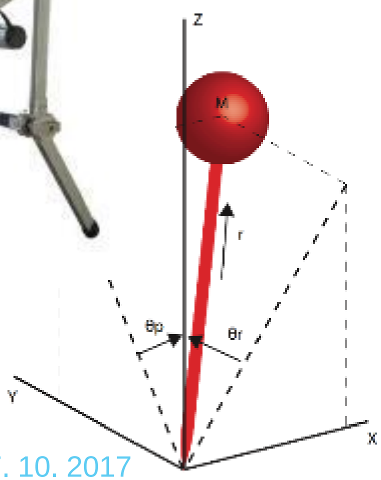
Distributed Control System (DCS) at the DCAI

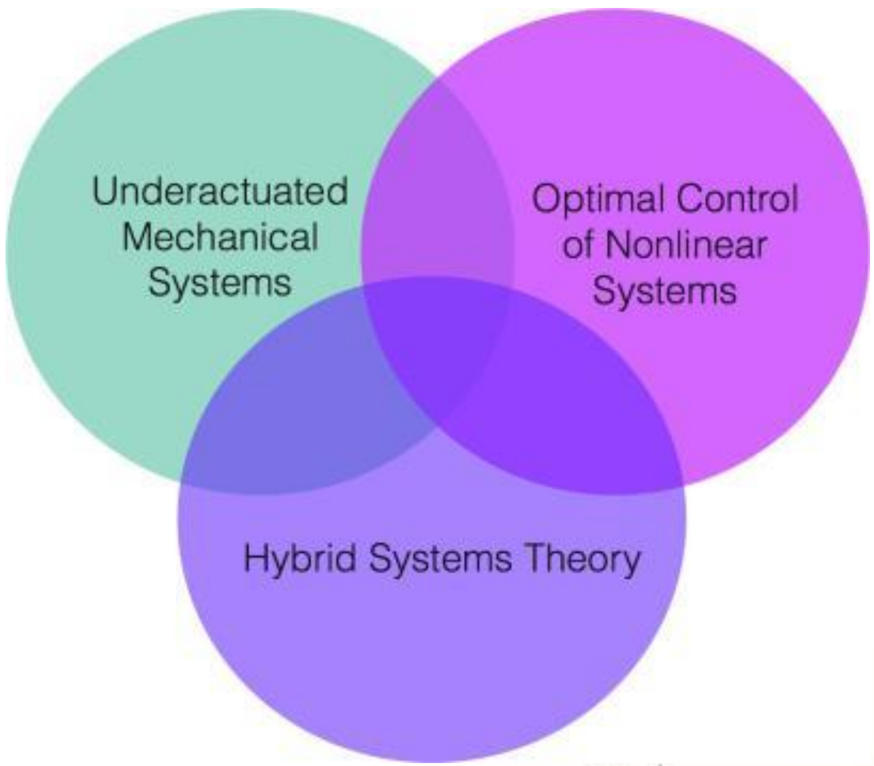
Single inverted pendulum Laboratory Model with the LSM

- Implementation into the DCS
- Simulation Model Identification
- Hybrid Control Design and Verification

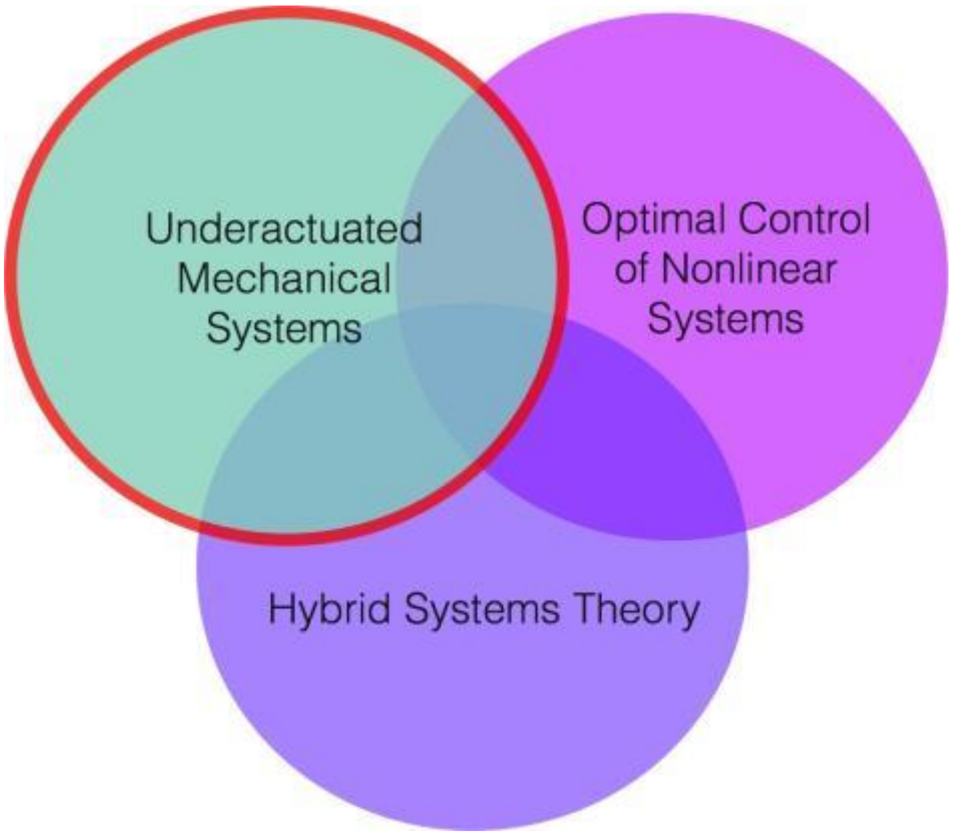
MODELING AND OPTIMAL CONTROL OF NONLINEAR UNDERACTUATED DYNAMICAL SYSTEMS (2011-2015)



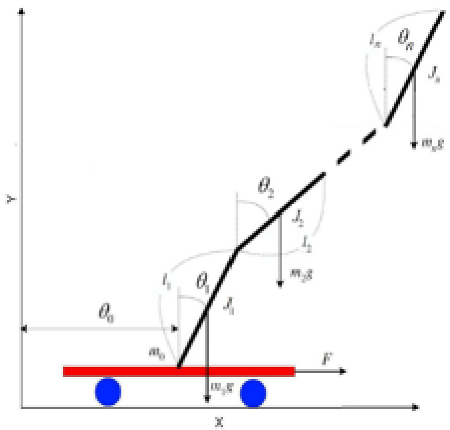


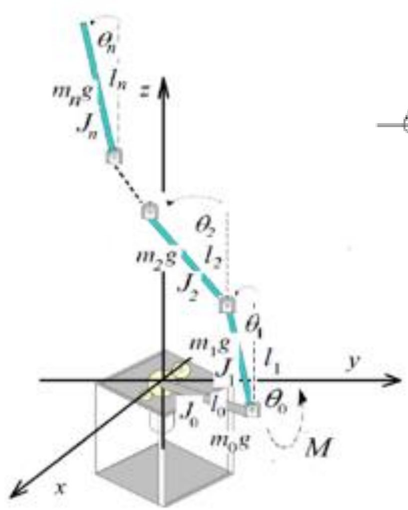


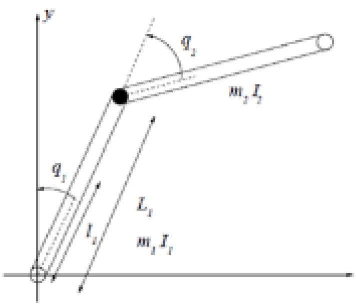
MODELING OF NONLINEAR UNDERACTUATED MECHANICAL SYSTEMS USING CLASSICAL MECHANICS

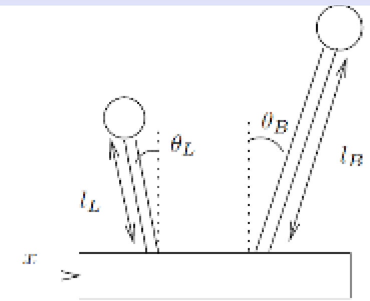


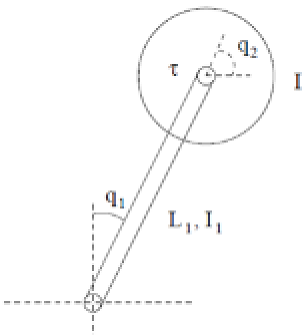
BENCHMARK UNDERACTUATED SYSTEMS

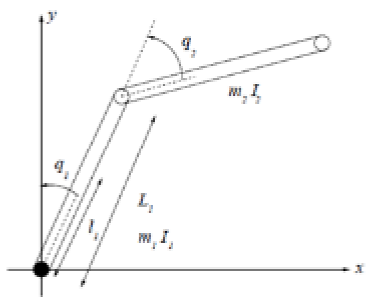


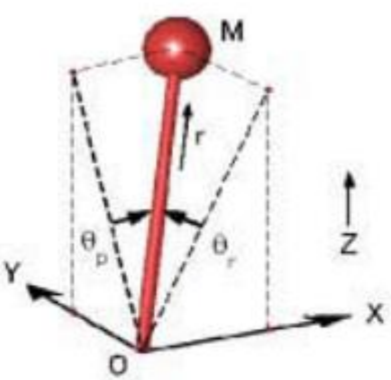


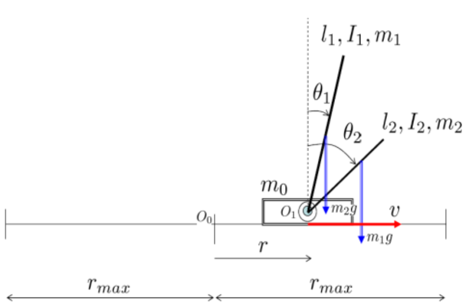


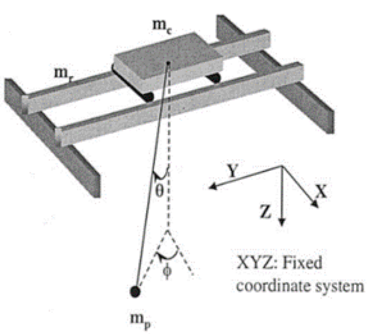


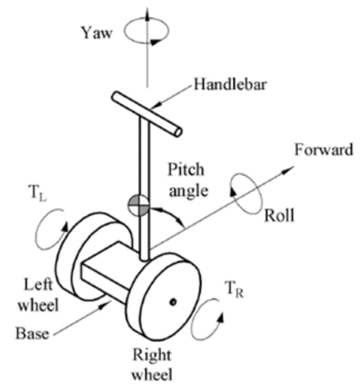












APPLICATION OF EULER-LAGRANGE EQUATIONS OF THE 2ND KIND IN MODELING OF A MULTI-BODY MECHANICAL SYSTEM

VECTOR OF GENERALIZED COORDINATES

$$\boldsymbol{\theta}(t) = \begin{pmatrix} \theta_1(t) & \theta_2(t) & \dots & \theta_k(t) \end{pmatrix}^T$$

Kinetic, potential and dissipative energies of a multi-body system:

$$E_K(t) = \sum_{i=1}^{k} E_{Ki}(t) \qquad E_P(t) = \sum_{i=1}^{k} E_{Pi}(t) \qquad D(t) = \sum_{i=1}^{k} D_i(t)$$

$$\mathcal{L}(t) = E_K\left(\boldsymbol{\theta}(t), \dot{\boldsymbol{\theta}}(t)\right) - E_P\left(\boldsymbol{\theta}(t)\right) \qquad D(t) = D\left(\boldsymbol{\theta}(t)\right)$$

LAGRANGE FUNCTION

DISSIPATIVE FUNCTION

$$\boxed{\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \mathcal{L}(t)}{\partial \dot{\boldsymbol{\theta}}(t)}\right) - \frac{\partial \mathcal{L}(t)}{\partial \boldsymbol{\theta}(t)} + \frac{\partial D(t)}{\partial \dot{\boldsymbol{\theta}}(t)} = \boldsymbol{Q}^*(t)}$$

EULER-LAGRANGE EQUATIONS OF THE 2ND KIND

- the process of deriving motion equations for a mechanical system is automated using symbolic software tools: **MATLAB**, **Maple**, **Mathematica**

FULLY ACTUATED VS. UNDERACTUATED SYSTEMS

$$\ddot{\boldsymbol{\theta}}(t) = \boldsymbol{f}(\boldsymbol{\theta}(t), \dot{\boldsymbol{\theta}}(t), \boldsymbol{u}(t), t)$$

GENERAL LAGRANGIAN NONLINEAR SYSTEM

$$\ddot{\boldsymbol{\theta}}(t) = \boldsymbol{f}_1\left(\boldsymbol{\theta}(t), \dot{\boldsymbol{\theta}}(t), t\right) + \boldsymbol{G}\left(\boldsymbol{\theta}(t), \dot{\boldsymbol{\theta}}(t), t\right)\boldsymbol{u}(t)$$

REARRANGEMENT INTO THE AFFINE FORM

$$\boxed{\boldsymbol{M}\left(\boldsymbol{\theta}(t)\right)\ddot{\boldsymbol{\theta}}(t) + \boldsymbol{N}\left(\boldsymbol{\theta}(t), \dot{\boldsymbol{\theta}}(t)\right)\dot{\boldsymbol{\theta}}(t) + \boldsymbol{R}\left(\boldsymbol{\theta}(t)\right) = \boldsymbol{V}(t)\boldsymbol{u}(t)}$$

REARRANGEMENT INTO THE STANDARD MINIMAL FORM

$$\ddot{\boldsymbol{\theta}}(t) = \left(\boldsymbol{M}\left(\boldsymbol{\theta}(t)\right)\right)^{-1}\left(\boldsymbol{V}(t)\boldsymbol{u}(t) - \boldsymbol{N}\left(\boldsymbol{\theta}(t), \dot{\boldsymbol{\theta}}(t)\right)\dot{\boldsymbol{\theta}}(t) - \boldsymbol{R}\left(\boldsymbol{\theta}(t)\right)\right)$$

$$\ddot{\boldsymbol{\theta}}(t) = \left(\boldsymbol{M}\left(\boldsymbol{\theta}(t)\right)\right)^{-1}\left(-\boldsymbol{N}\left(\boldsymbol{\theta}(t), \dot{\boldsymbol{\theta}}(t)\right)\dot{\boldsymbol{\theta}}(t) - \boldsymbol{R}\left(\boldsymbol{\theta}(t)\right)\right) + \left(\boldsymbol{M}\left(\boldsymbol{\theta}(t)\right)\right)^{-1}\boldsymbol{V}(t)\boldsymbol{u}(t)$$

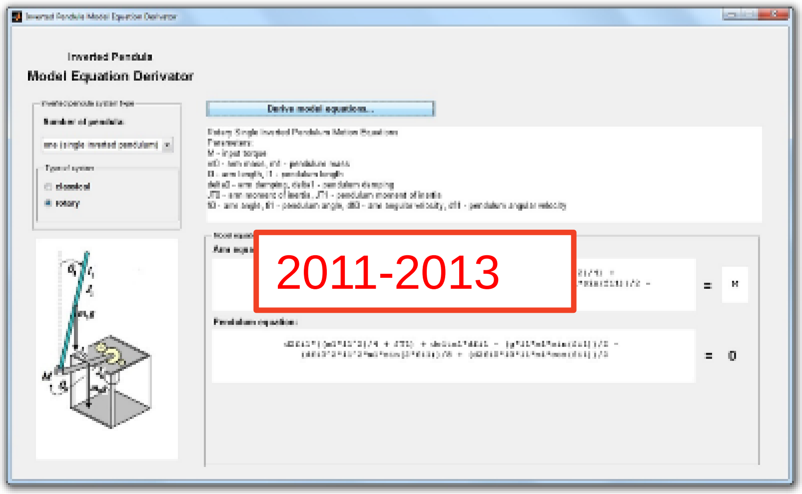
FULLY ACTUATED SYSTEM

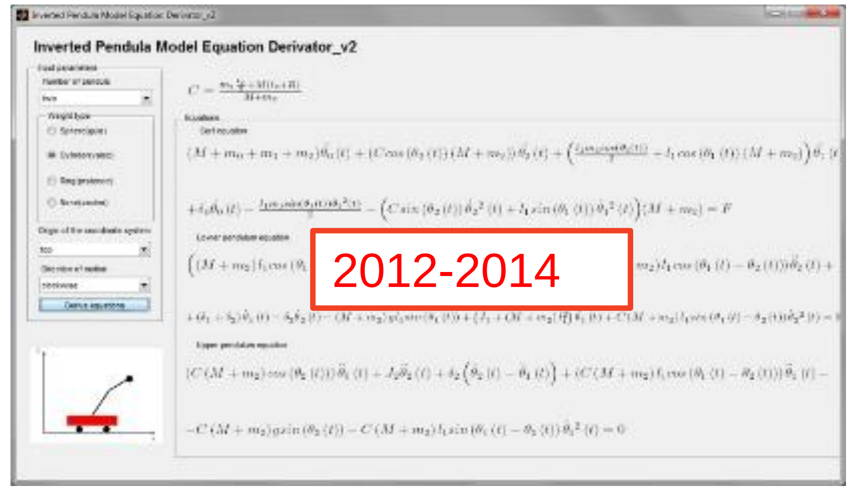
$$rank\left(\boldsymbol{G}\left(\boldsymbol{\theta}(t), \dot{\boldsymbol{\theta}}(t), t\right)\right) = rank\left(\boldsymbol{V}\left(\boldsymbol{\theta}(t)\right)\right) = \dim\left(\boldsymbol{\theta}(t)\right)$$

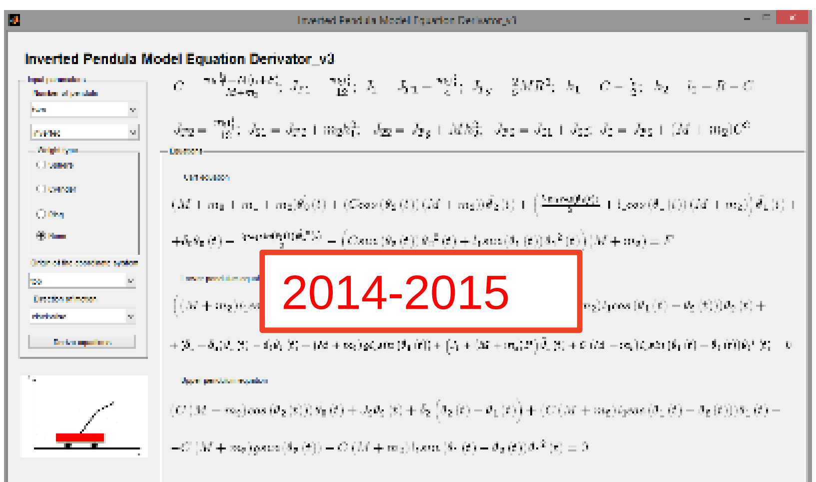
UNDERACTUATED SYSTEM

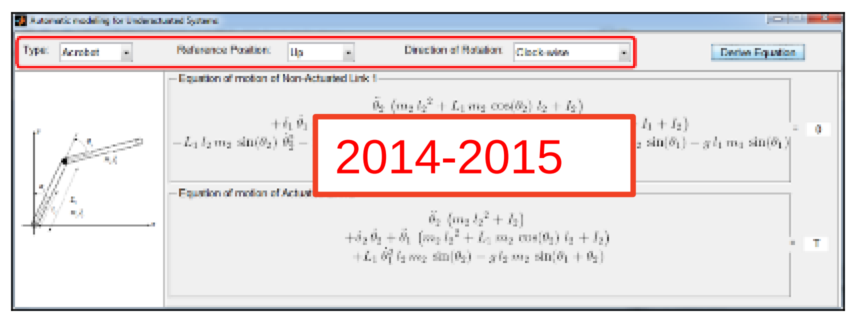
$$\boxed{rank\left(\boldsymbol{G}\left(\boldsymbol{\theta}(t), \dot{\boldsymbol{\theta}}(t), t\right)\right) = rank\left(\boldsymbol{V}\left(\boldsymbol{\theta}(t)\right)\right) < \dim\left(\boldsymbol{\theta}(t)\right)}$$

AUTOMATIC DERIVATION OF MOTION EQUATIONS FOR BENCHMARK UNDERACTUATED SYSTEMS (2011 – 2015)







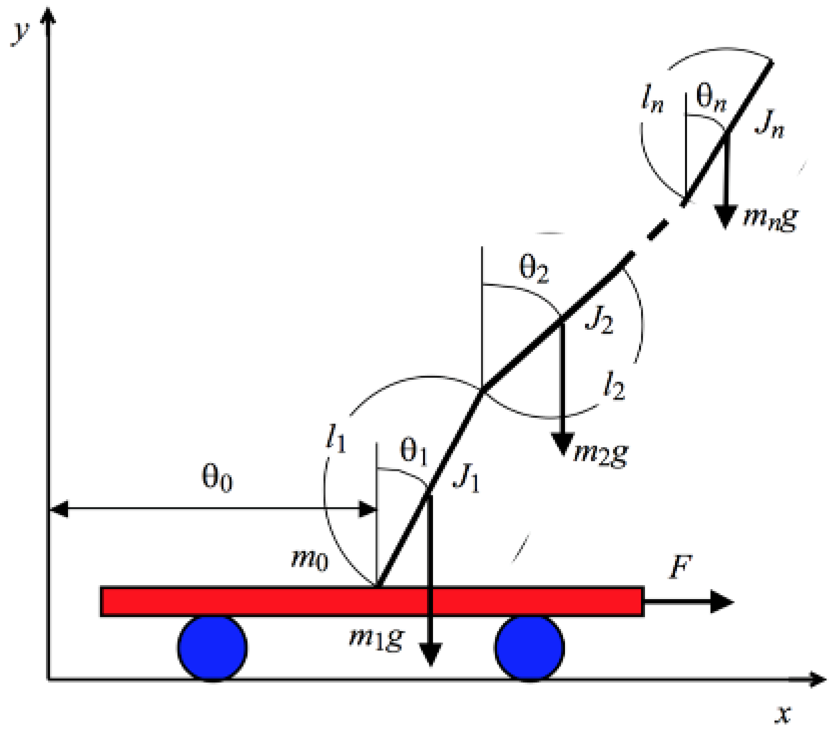


MATHEMATICAL MODELING OF INVERTED PENDULUM SYSTEMS
A GENERALIZED APPROACH

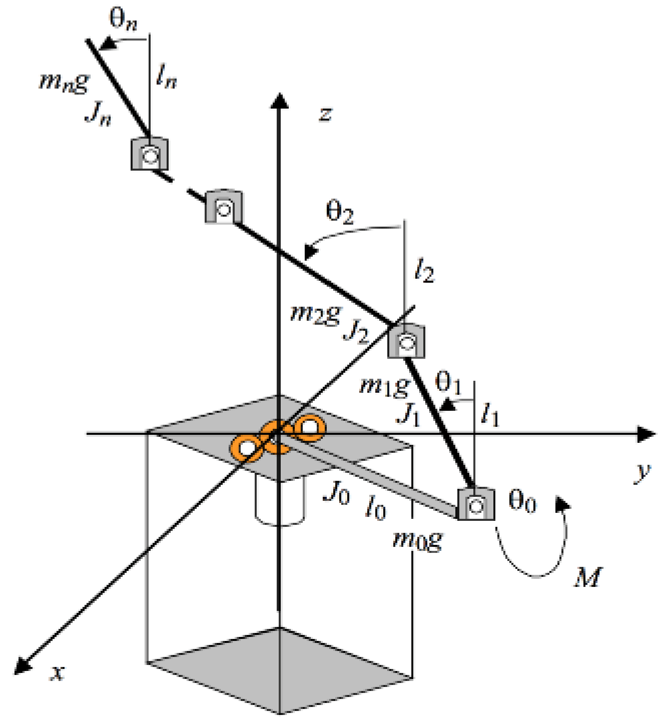
VECTOR OF GENERALIZED COORDINATES:

$$\boldsymbol{\theta}(t) = \begin{pmatrix}\theta_0(t) & \theta_1(t) & \ldots & \theta_n(t)\end{pmatrix}^T$$

cart/arm position   pendulum angles



GENERALIZED SYSTEM OF *N* CLASSICAL INVERTED PENDULA



GENERALIZED SYSTEM OF *N* ROTARY INVERTED PENDULA

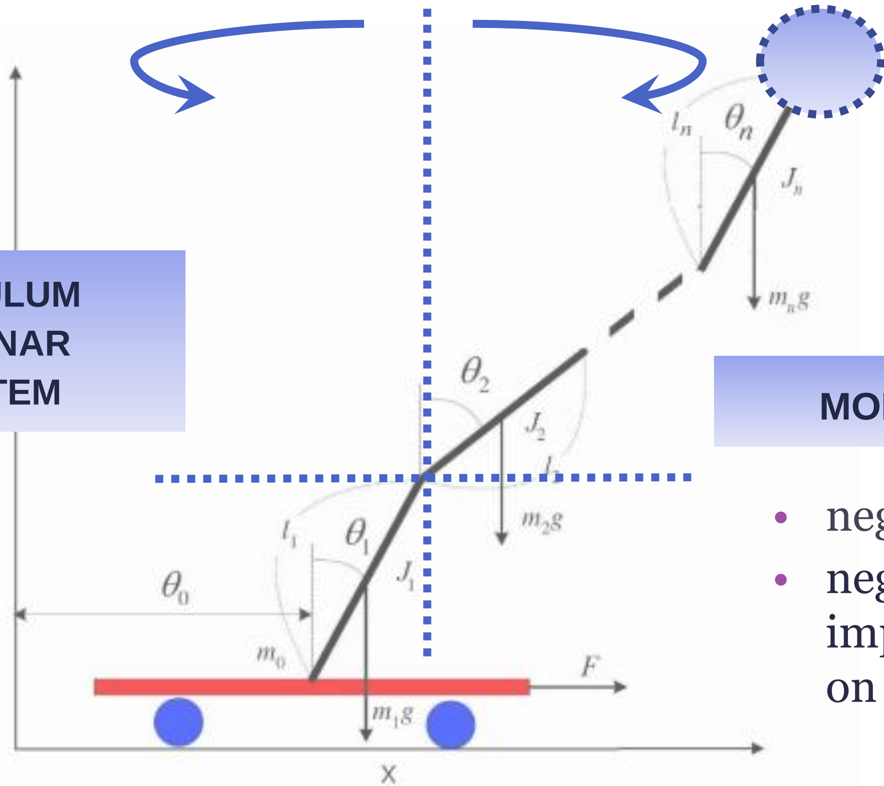
EXPANSIONS OF THE GENERALIZED INVERTED PENDULUM SYSTEM

REFERENCE DIRECTION OF PENDULUM ROTATION

- clockwise
- counterclockwise

WEIGHT ATTACHED TO THE END OF *N*-TH PENDULUM LINK

- sphere
- cylinder
- disk



REFERENCE PENDULUM POSITION IN A PLANAR COORDINATE SYSTEM

- top
- right
- down
- left

MODEL SIMPLIFICATION

- neglected friction
- neglected backward impact of the pendulum on the cart

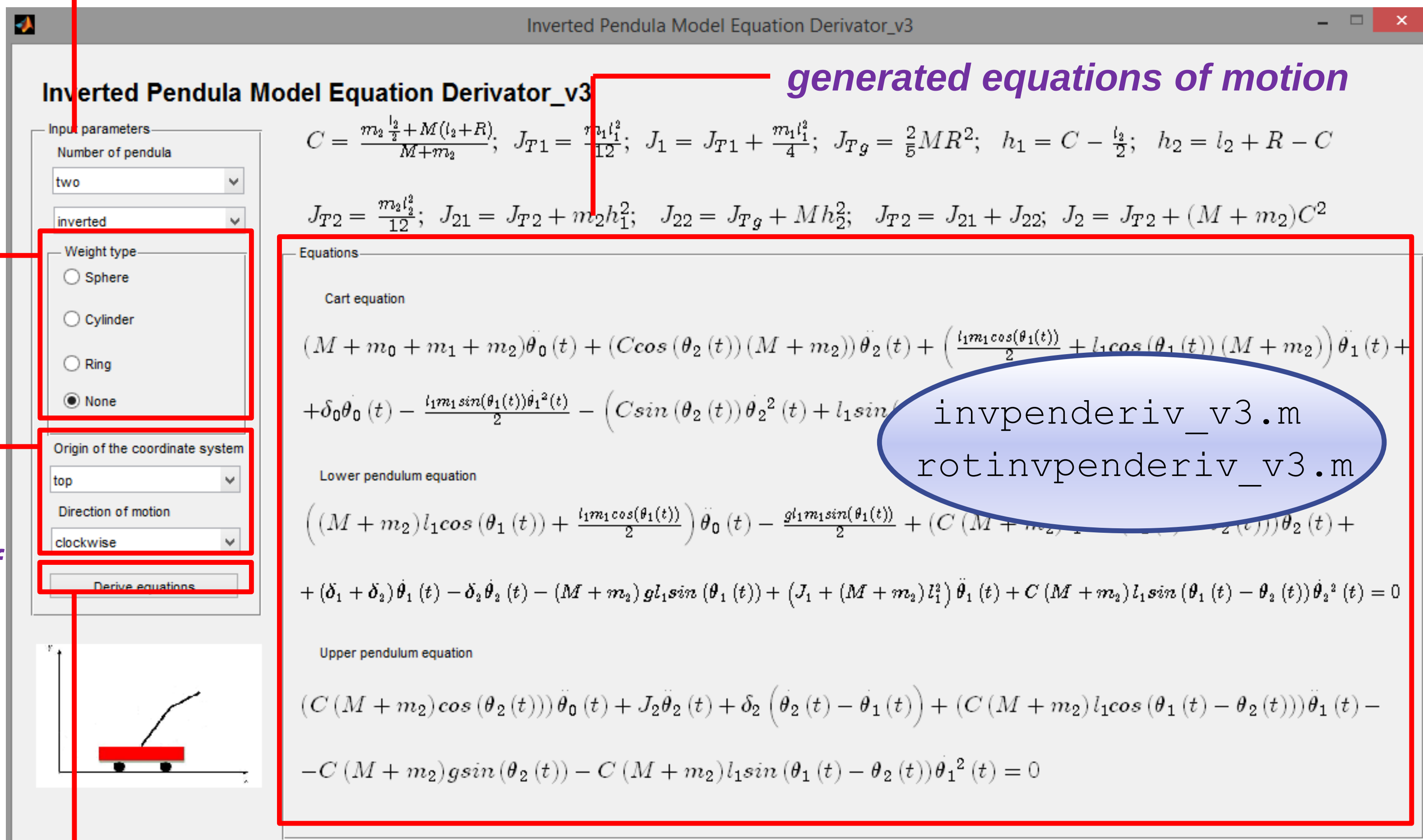
INVERTED PENDULA MODEL EQUATION DERIVATOR_V3 (AUTOMATIC DERIVATION OF MOTION EQUATIONS)

selects the system type & number of pendulum links

generated equations of motion

attached weight type

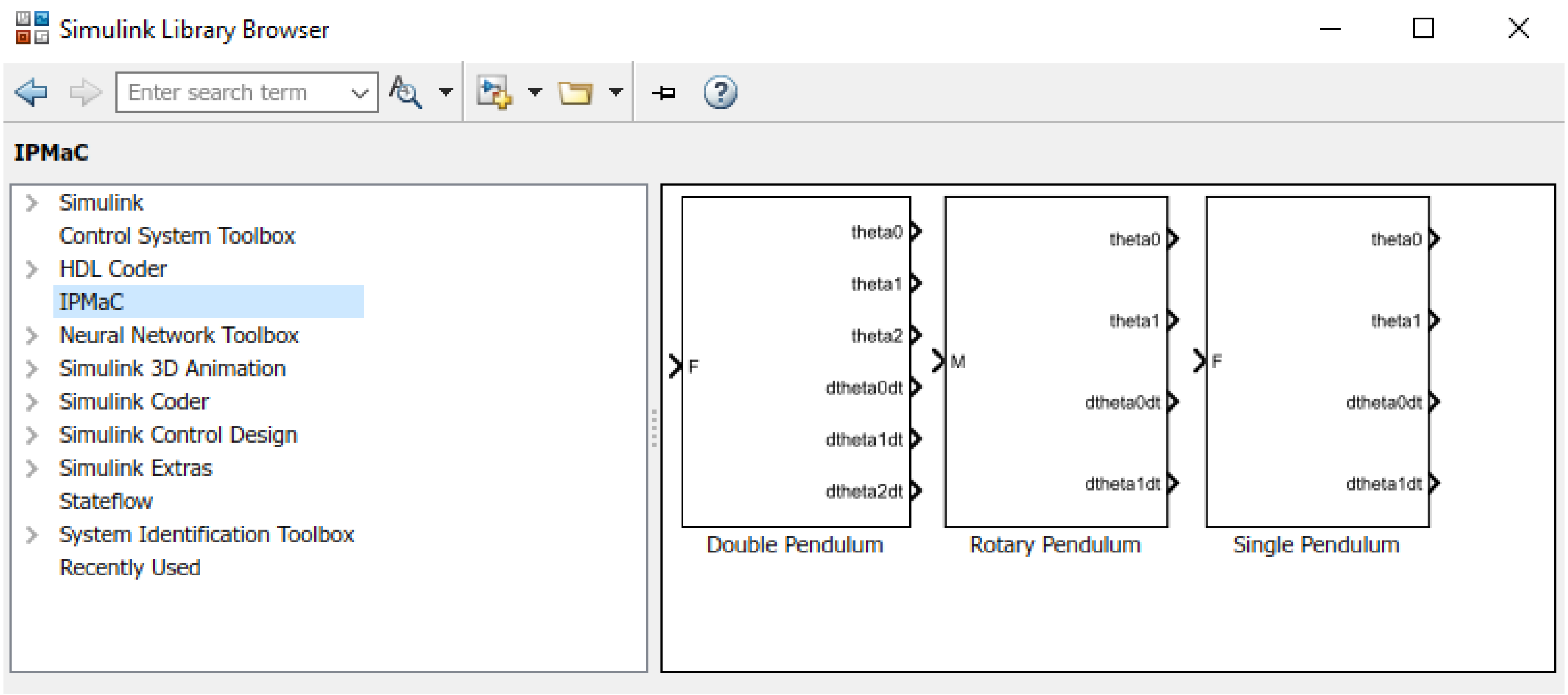
reference pendulum position & direction of pendulum rotation



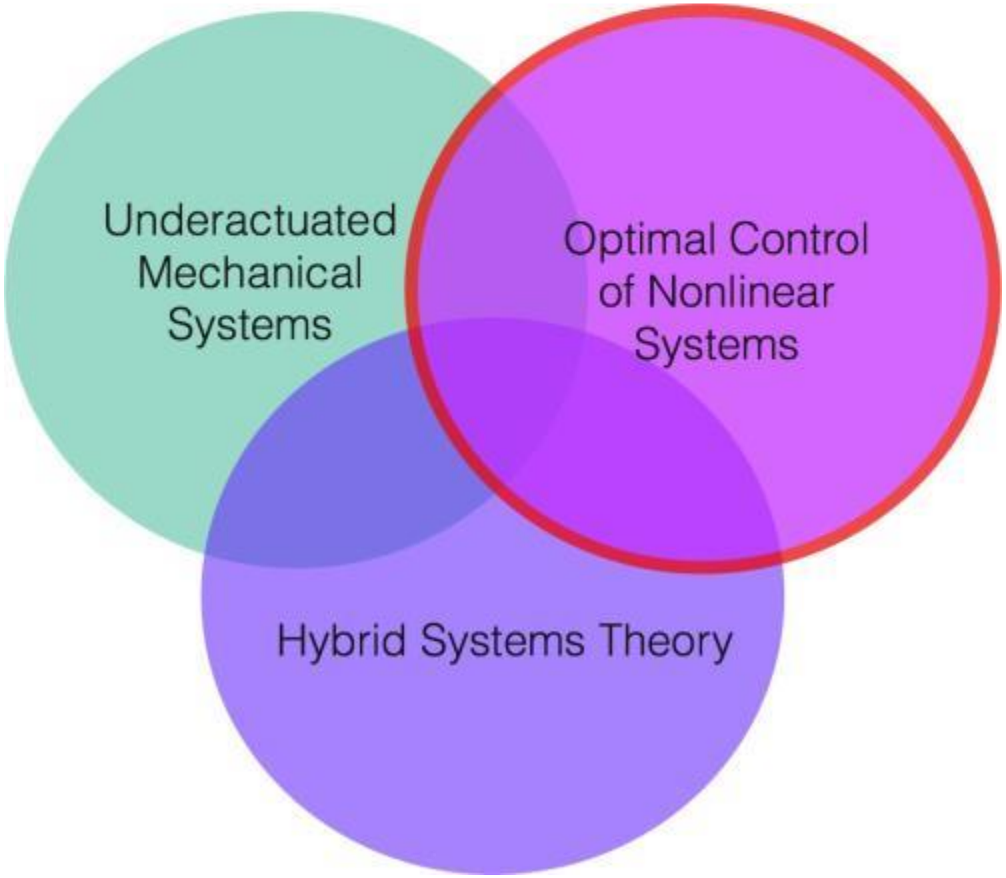
initiates mathematical model derivation

INVERTED PENDULA MODELING AND CONTROL

LIBRARY BLOCKS – INVERTED PENDULA MODELS



OPTIMAL CONTROL OF NONLINEAR UNDERACTUATED MECHANICAL SYSTEMS



OPTIMAL CONTROL PROBLEM FOR NONLINEAR DYNAMICAL SYSTEMS

NONLINEAR DYNAMICAL SYSTEM

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{f}\left(\boldsymbol{x}(t), \boldsymbol{u}(t), t\right)$$
$$\boldsymbol{y}(t) = \boldsymbol{g}\left(\boldsymbol{x}(t), \boldsymbol{u}(t), t\right)$$
$$\boldsymbol{x}(t_0) = \boldsymbol{x}_0$$

underactuated system with k DoFs

$$\boldsymbol{x}(t) \in \mathbb{R}^{2k\times 1}$$
$$\boldsymbol{y}(t) \in \mathbb{R}^{m\times 1}$$
$$\boldsymbol{u}(t) \in \mathbb{R}^{s\times 1}$$

OPTIMAL STATE-FEEDBACK CONTROL

$$J\left(\boldsymbol{x}_0, \boldsymbol{u}(t), t_0\right) = \phi\left(\boldsymbol{x}(t_f)\right) + \int_{t_0}^{t_f} V_{CF}\left(\boldsymbol{x}(t), \boldsymbol{u}(t), t\right) dt \rightarrow min, \quad t \in [t_0, t_f]$$

- **underactuated** systems are more difficult to control than **fully actuated systems**:
  - useful properties of fully actuated systems (*feedback linearizability, linear parametrizability)* are lost
  - adverse properties typical of underactuated systems emerge – *higher relative degree, nonminimal phase dynamics*

State-space Description of Nonlinear Underactuated Dynamical Systems

$$\boldsymbol{x}(t) = \begin{pmatrix}\boldsymbol{\theta}(t) & \dot{\boldsymbol{\theta}}(t)\end{pmatrix}^T = \begin{pmatrix} x_1(t) & x_2(t) & \dots & x_{2k}(t)\end{pmatrix}^T$$

State vector sized 2k

$$\ddot{\boldsymbol{\theta}}(t) = \left(\boldsymbol{M}\left(\boldsymbol{\theta}(t)\right)\right)^{-1}\left(\boldsymbol{V}(t)\boldsymbol{u}(t) - \boldsymbol{N}\left(\boldsymbol{\theta}(t),\dot{\boldsymbol{\theta}}(t)\right)\dot{\boldsymbol{\theta}}(t) - \boldsymbol{R}\left(\boldsymbol{\theta}(t)\right)\right)$$

$$\boldsymbol{y}(t) = \boldsymbol{h}\left(\boldsymbol{x}(t),\boldsymbol{u}(t),t\right) = \boldsymbol{C}\boldsymbol{x}(t) + \boldsymbol{D}\boldsymbol{u}(t)$$

$$\boldsymbol{C} \in \mathbf{R}^{m\times 2k} \qquad \boldsymbol{D} \in \mathbf{R}^{m\times s}$$

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{f}\left(\boldsymbol{x}(t),\boldsymbol{u}(t),t\right) = \begin{pmatrix} 0 & \boldsymbol{I} \\ 0 & -\boldsymbol{M}^{-1}\boldsymbol{N}\end{pmatrix}\boldsymbol{x}(t) + \begin{pmatrix} 0 \\ -\boldsymbol{M}^{-1}\boldsymbol{R}\end{pmatrix} + \begin{pmatrix} 0 \\ -\boldsymbol{M}^{-1}\boldsymbol{V}\end{pmatrix}\boldsymbol{u}(t)$$

Nonlinear state-space representation of a general underactuated system

Linear Approximation of Nonlinear Underactuated Dynamical Systems

if the linear approximation of the nonlinear system is known around the given equilibrium point, optimal control techniques designed for linear systems can be applied to control the original nonlinear system

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{f}\left(\boldsymbol{x}(t), \boldsymbol{u}(t), t\right)$$

$$f_i^*\left(\boldsymbol{x}(t), \boldsymbol{u}(t), t\right) \approx f_i\left(\boldsymbol{x}_S, \boldsymbol{u}_S\right) + \sum_{j=1}^{2k} \left.\frac{\partial f_i\left(\boldsymbol{x}(t), \boldsymbol{u}(t), t\right)}{\partial x_j(t)}\right|_{\boldsymbol{x}_S, \boldsymbol{u}_S} \left(x_j(t) - x_{jS}\right) + \sum_{j=1}^{s} \left.\frac{\partial f_i\left(\boldsymbol{x}(t), \boldsymbol{u}(t), t\right)}{\partial u_j(t)}\right|_{\boldsymbol{x}_S, \boldsymbol{u}_S} \left(u_j(t) - u_{jS}\right)$$

$$\left.\begin{aligned} \boldsymbol{A} &= \left.\left(\frac{\partial \boldsymbol{f}\left(\boldsymbol{x}(t), \boldsymbol{u}(t), t\right)}{\partial \boldsymbol{x}(t)}\right)\right|_{\boldsymbol{x}_S, \boldsymbol{u}_S} \in \mathbb{R}^{2k \times 2k} \\ \boldsymbol{B} &= \left.\left(\frac{\partial \boldsymbol{f}\left(\boldsymbol{x}(t), \boldsymbol{u}(t), t\right)}{\partial \boldsymbol{u}(t)}\right)\right|_{\boldsymbol{x}_S, \boldsymbol{u}_S} \in \mathbb{R}^{2k \times s} \end{aligned}\right\}$$

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\Delta\boldsymbol{x}(t) + \boldsymbol{B}\Delta\boldsymbol{u}(t)$$

State Equation in Deviation Form

$$\boldsymbol{x}_S = \boldsymbol{0}^T$$

$$\begin{aligned} \dot{\boldsymbol{x}}(t) &= \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t) \\ \dot{\boldsymbol{y}}(t) &= \boldsymbol{C}\boldsymbol{y}(t) + \boldsymbol{D}\boldsymbol{u}(t) \end{aligned} \longrightarrow \begin{aligned} \boldsymbol{x}(i+1) &= \boldsymbol{F}\boldsymbol{x}(i) + \boldsymbol{G}\boldsymbol{u}(i) \\ \boldsymbol{y}(i) &= \boldsymbol{C}\boldsymbol{x}(i) + \boldsymbol{D}\boldsymbol{u}(i) \end{aligned}$$

Linear Quadratic Regulator (LQR) Design Based on the Continuous-Time and Discrete-Time State-Space Model of the Linear System

$$\boxed{J_{LQ}(t) = \int_0^\infty \left(\boldsymbol{x}^T(t)\boldsymbol{Q}_{LQ}\boldsymbol{x}(t) + \boldsymbol{u}_R^T(t)\boldsymbol{R}_{LQ}\boldsymbol{u}_R(t)\right) dt}$$

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t)$$

$$\dot{\boldsymbol{y}}(t) = \boldsymbol{C}\boldsymbol{y}(t) + \boldsymbol{D}\boldsymbol{u}(t)$$

$$\boldsymbol{A}^T\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A} - \boldsymbol{P}\boldsymbol{B}\boldsymbol{R}_{LQ}^{-1}\boldsymbol{B}^T\boldsymbol{P} + \boldsymbol{Q}_{LQ} = \boldsymbol{0}$$

Continuous-time algebraic Riccati equation

$$\boxed{J_{LQ}(i) = \sum_{i=0}^{N-1} \left[\boldsymbol{x}^T(i)\boldsymbol{Q}_{LQ}\boldsymbol{x}(i) + \boldsymbol{u}_R^T(i)\boldsymbol{R}_{LQ}\boldsymbol{u}_R(i)\right]}$$

$$\boldsymbol{x}(i+1) = \boldsymbol{F}\boldsymbol{x}(i) + \boldsymbol{G}\boldsymbol{u}(i)$$

$$\boldsymbol{y}(i) = \boldsymbol{C}\boldsymbol{x}(i) + \boldsymbol{D}\boldsymbol{u}(i)$$

$$\boldsymbol{F}^T\boldsymbol{P}\boldsymbol{F} - \boldsymbol{F}^T\boldsymbol{P}\boldsymbol{G}\left(\boldsymbol{R}_{LQ} + \boldsymbol{G}^T\boldsymbol{P}\boldsymbol{G}\right)^{-1}\boldsymbol{G}^T\boldsymbol{P}\boldsymbol{F} + \boldsymbol{Q}_{LQ} = 0$$

Discrete-time algebraic Riccati equation

$$\boldsymbol{K_{LQ}} = \boldsymbol{R}_{LQ}^{-1}\boldsymbol{B}^T\boldsymbol{P},$$

Optimal state-feedback gain matrix

$$\boldsymbol{K_{LQ}} = \boldsymbol{R}_{LQ}^{-1}\boldsymbol{B}^T\boldsymbol{P},$$

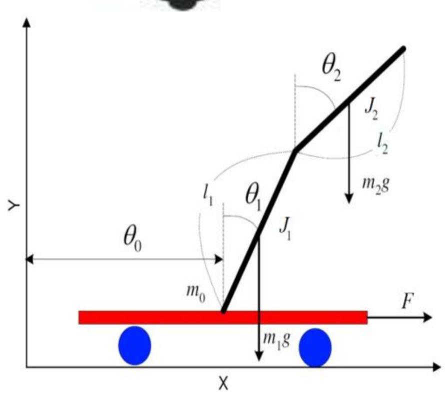
$$\boldsymbol{u}_R(t) = -\boldsymbol{K_{LQ}}\boldsymbol{x}(t)$$

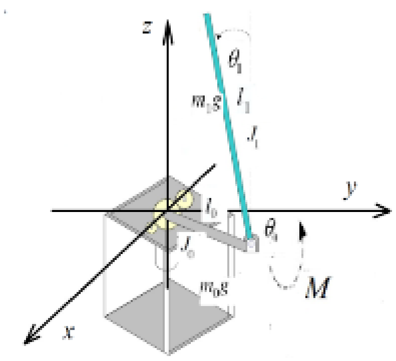
$$\boldsymbol{u}_R(i) = -\boldsymbol{K_{LQ}}(i)\boldsymbol{x}(i),$$

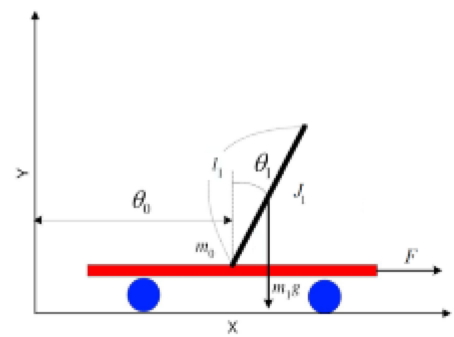
State-feedback gain matrix

Application of Optimal Control Techniques in State-feedback Control of Inverted Pendulum Systems as Simulation Models

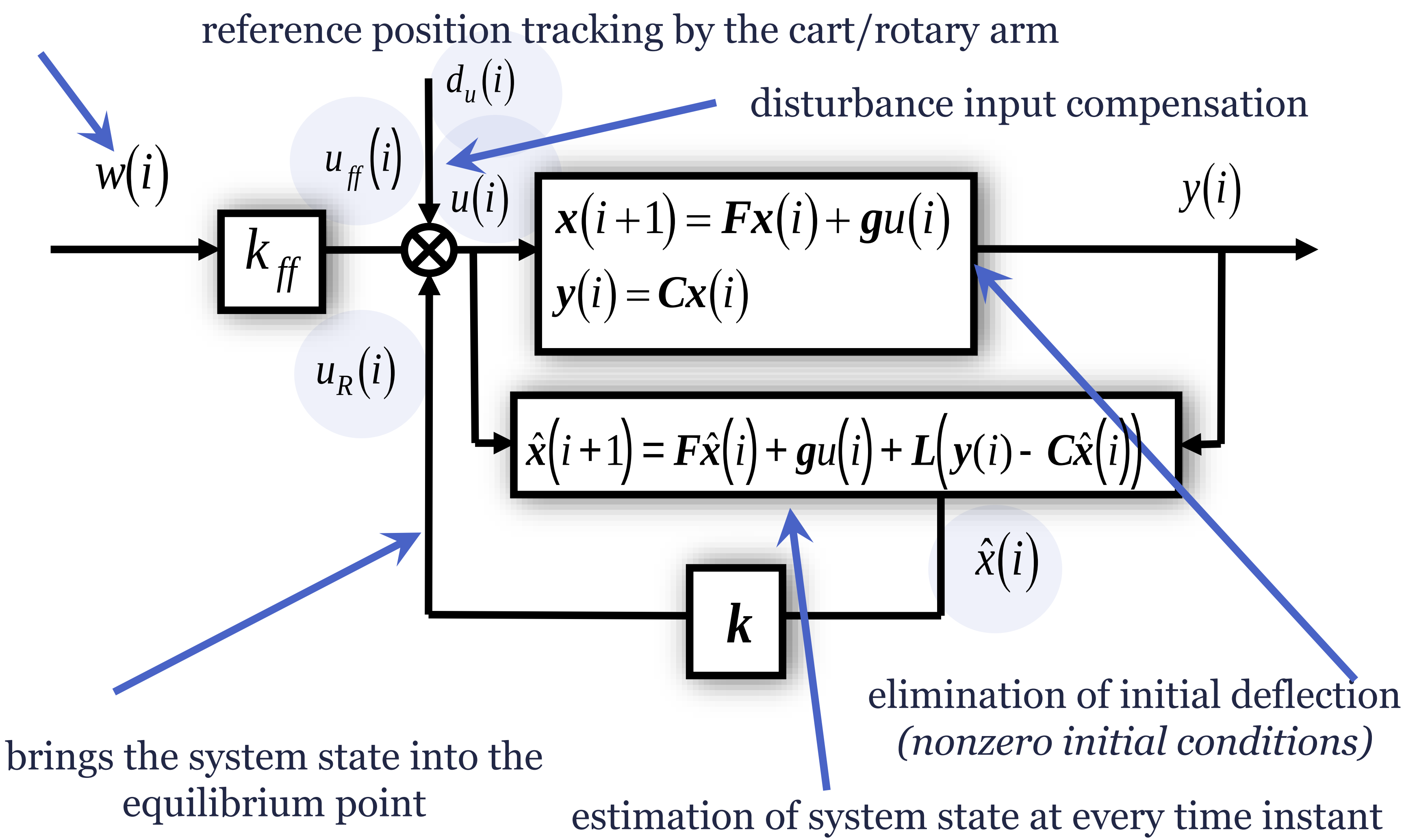
- **the aim is to design a stabilizing *state-feedback control law* which fulfills the principal control objective for inverted pendulum systems:**
  - stabilization of all pendulum links in the vertical upright (inverted) position (= unstable equilibrium)
- **and at the same time ensures that:**
  - the *initial deflection* of each pendulum link is eliminated
  - a time-constrained or permanent disturbance input signal is compensated
  - the cart/rotary arm tracks a predefined reference trajectory while keeping the pendulum links in the upright equilibrium all the time



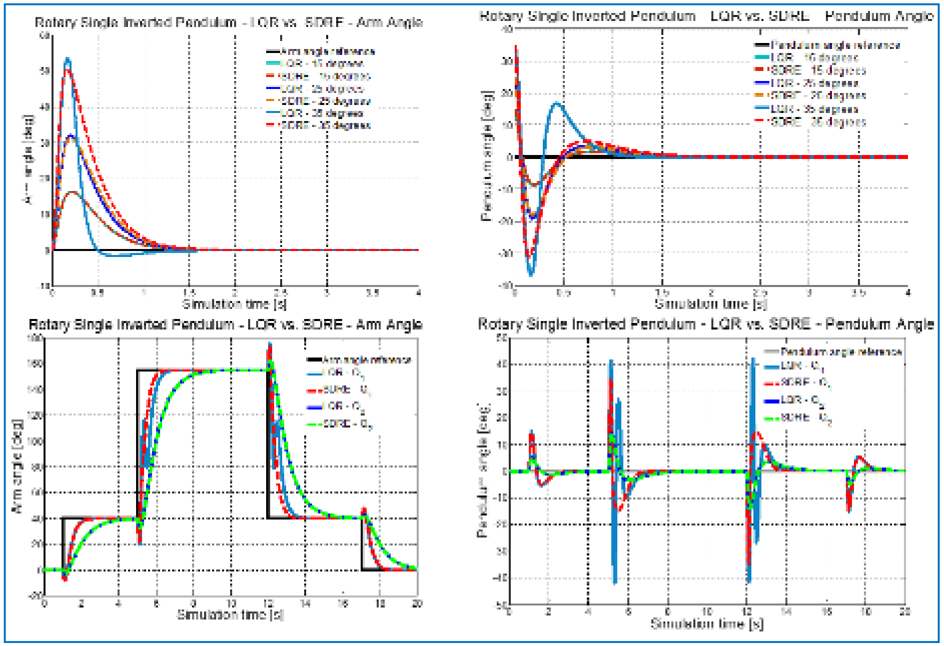




State-feedback control with feedforward gain & discrete-time state observer - block scheme

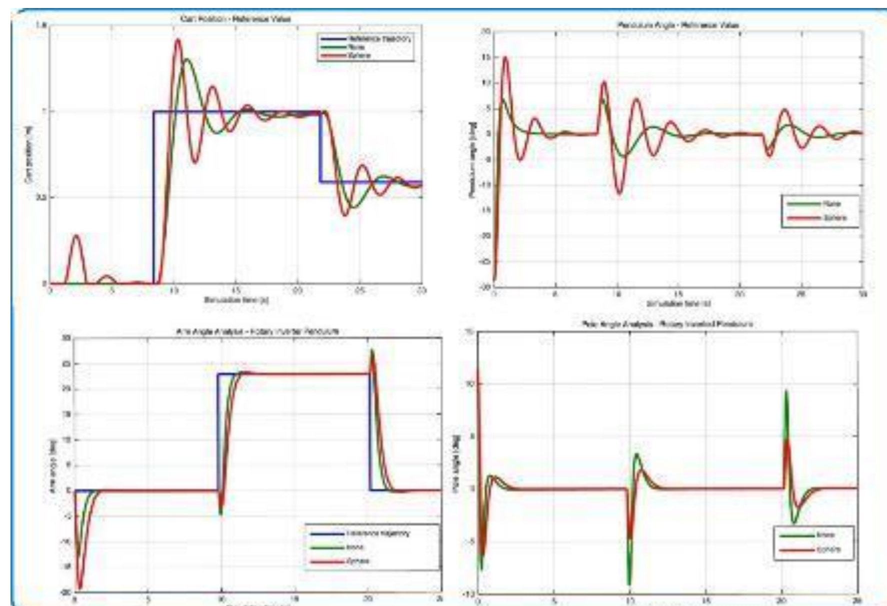


EXAMPLE RESULTS – OPTIMAL CONTROL OF UNDERACTUATED SYSTEMS



– stabilization of a rotary single inverted pendulum system in the upright unstable position for *a) initial deflection control b) reference command tracking by the rotary arm* using

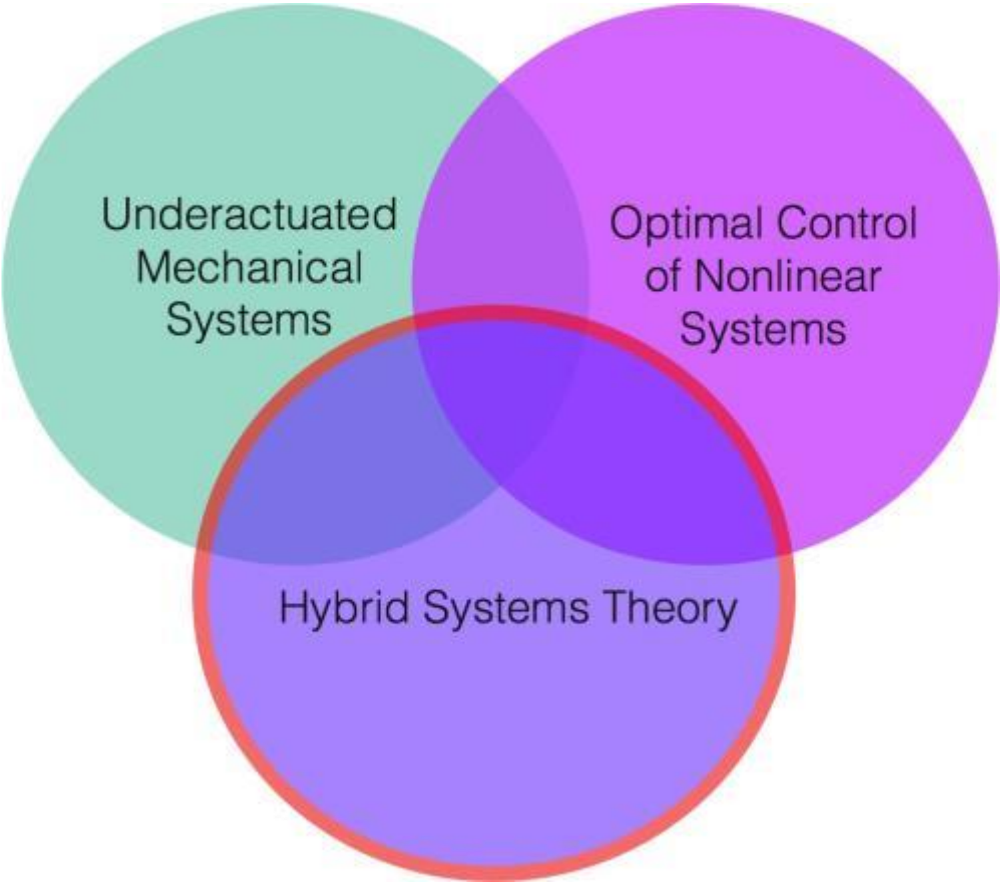
- ***LQR***
- ***State-dependent Riccati equation method***



– stabilization of a classical/rotary single inverted pendulum system in the upright unstable position for *reference command tracking by the rotary arm*, using

- ***Model Predictive Control (MPC)***

Hybrid Control Of Nonlinear Underactuated Mechanical Systems

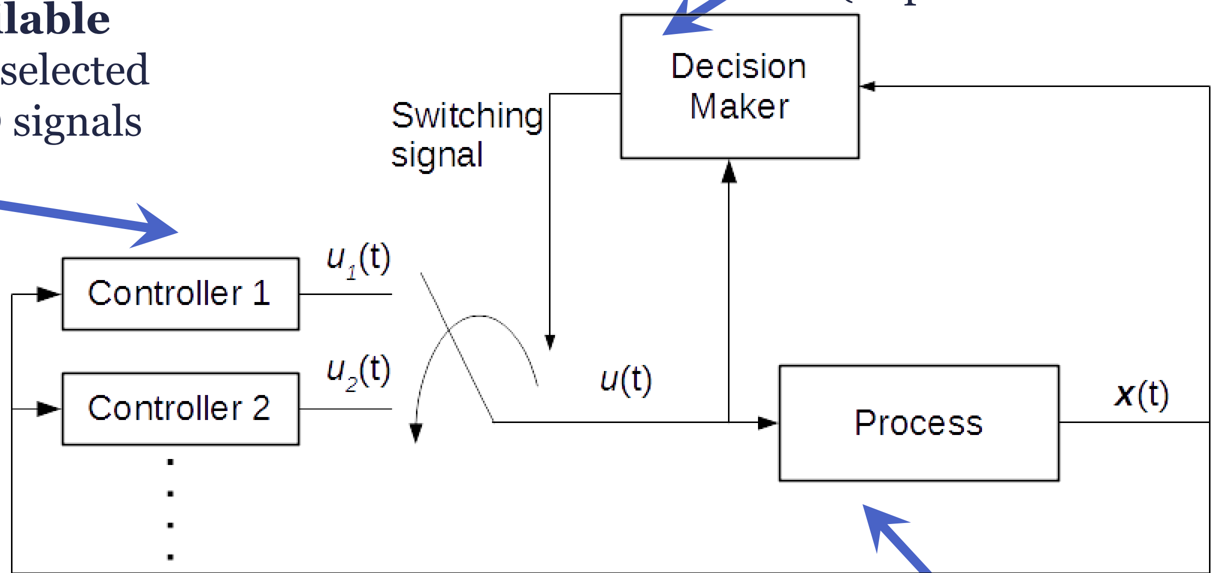


PRINCIPLES OF *SWITCHING/HYBRID CONTROL*

- *switched/hybrid closed-loop system* as an alternative to continuous control law (*Brockett condition* not met)

supervisory mechanism
(implements switching logic)

one of the **available control laws** is selected based on the I/O signals



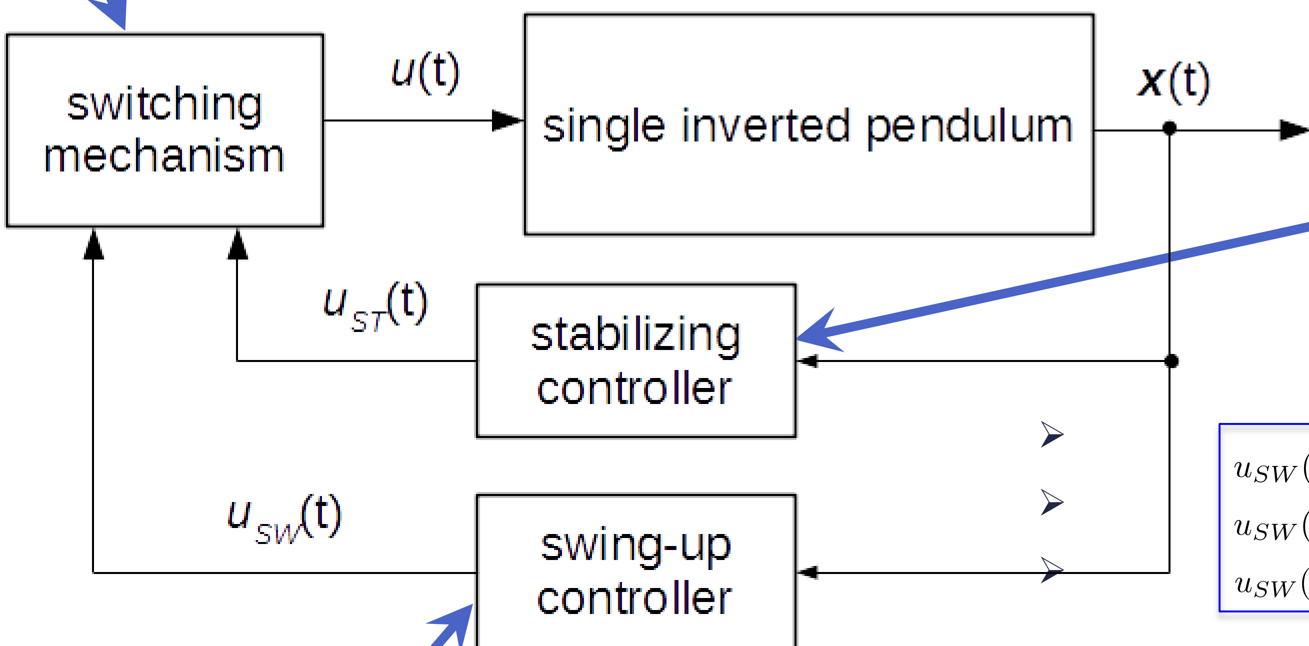
controlled system

HYBRID CONTROL STRUCTURE DESIGN
FOR INVERTED PENDULUM SWING-UP AND STABILIZATION

transition/switching mechanism

state-feedback balancing controller

- **LQR**
- **MPC**
- **SDRE**



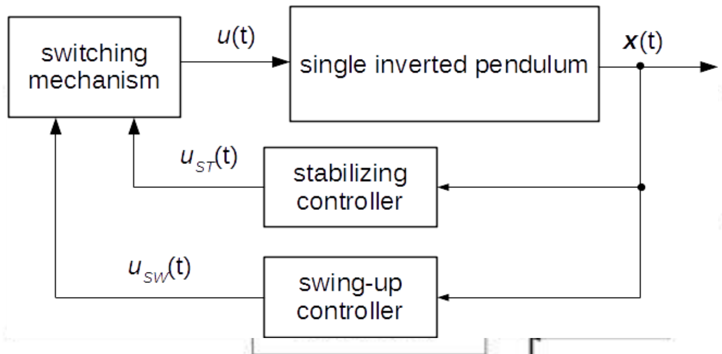
$$u_{SW}(t) = u_m \mathrm{sgn}\left(\dot{\theta}_1(t)|\theta_1(t)|\right)$$
$$u_{SW}(t) = u_m \mathrm{sgn}\left(\dot{\theta}_1(t)\right)$$
$$u_{SW}(t) = -u_m \mathrm{sgn}\left(\dot{\theta}_1(t)\cos\theta_1(t)\right)$$

swing-up controller

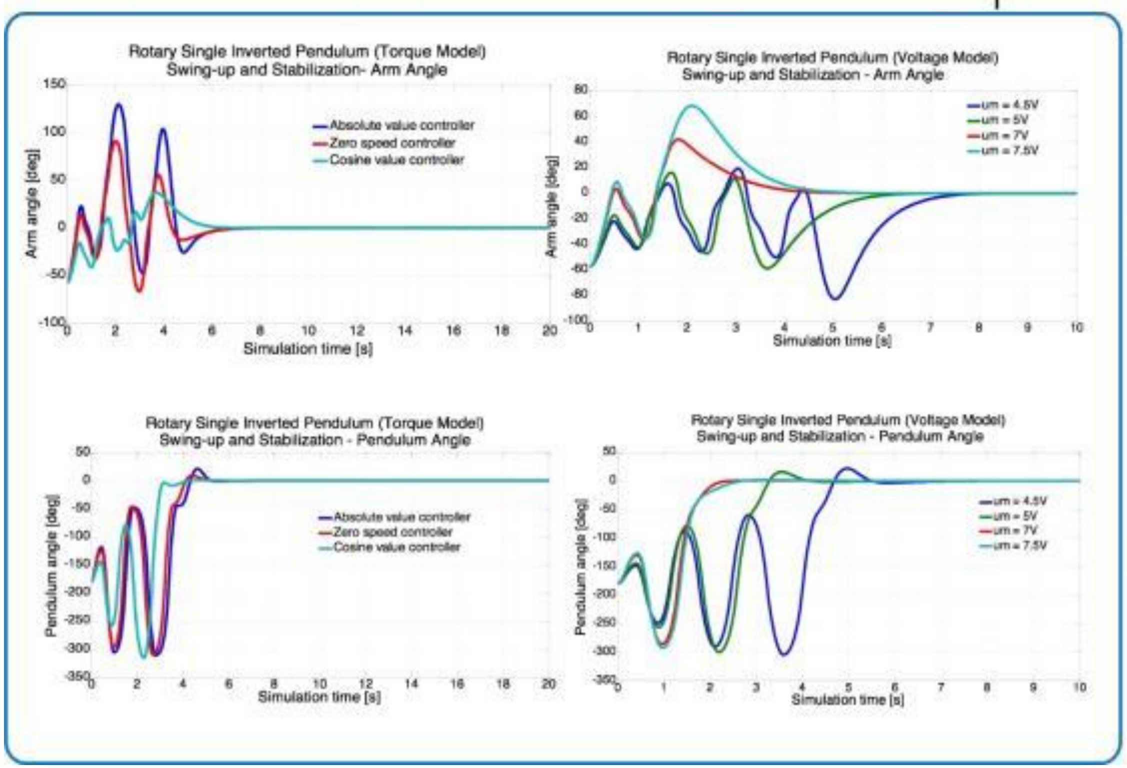
- **energy-based methods**
- **partial feedback linearization**

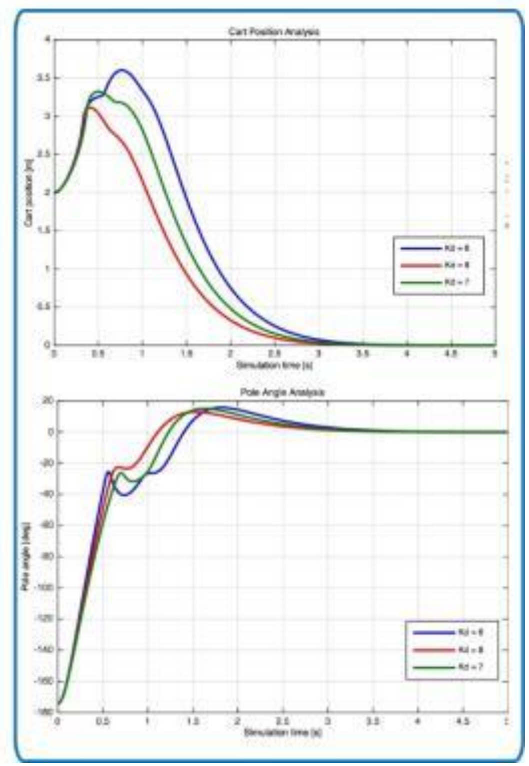
EXAMPLE RESULTS – HYBRID CONTROL STRUCTURE DESIGN FOR INVERTED PENDULUM SWING-UP AND STABILIZATION

swing-up: energy-based control
stabilization: LQR based on the discrete-time linear state-space model



swing-up: partial feedback linearization
stabilization: LQR based on the discrete-time linear state-space model

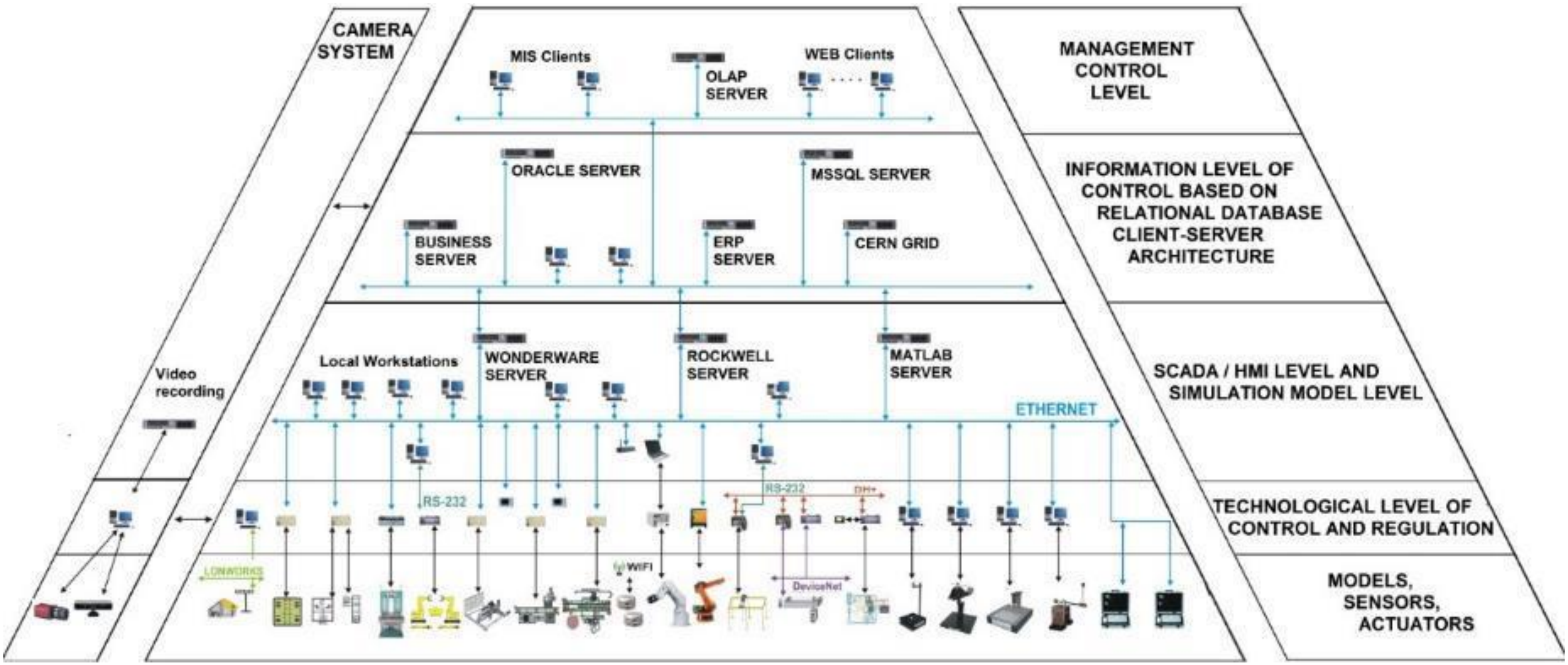




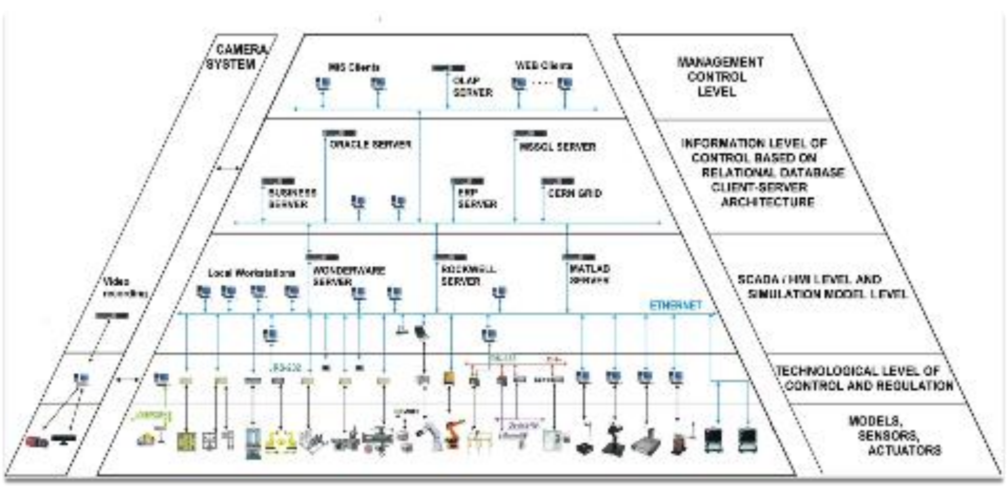
Single Inverted Pendulum Laboratory Model with the Linear Synchronous Motor (LSM):

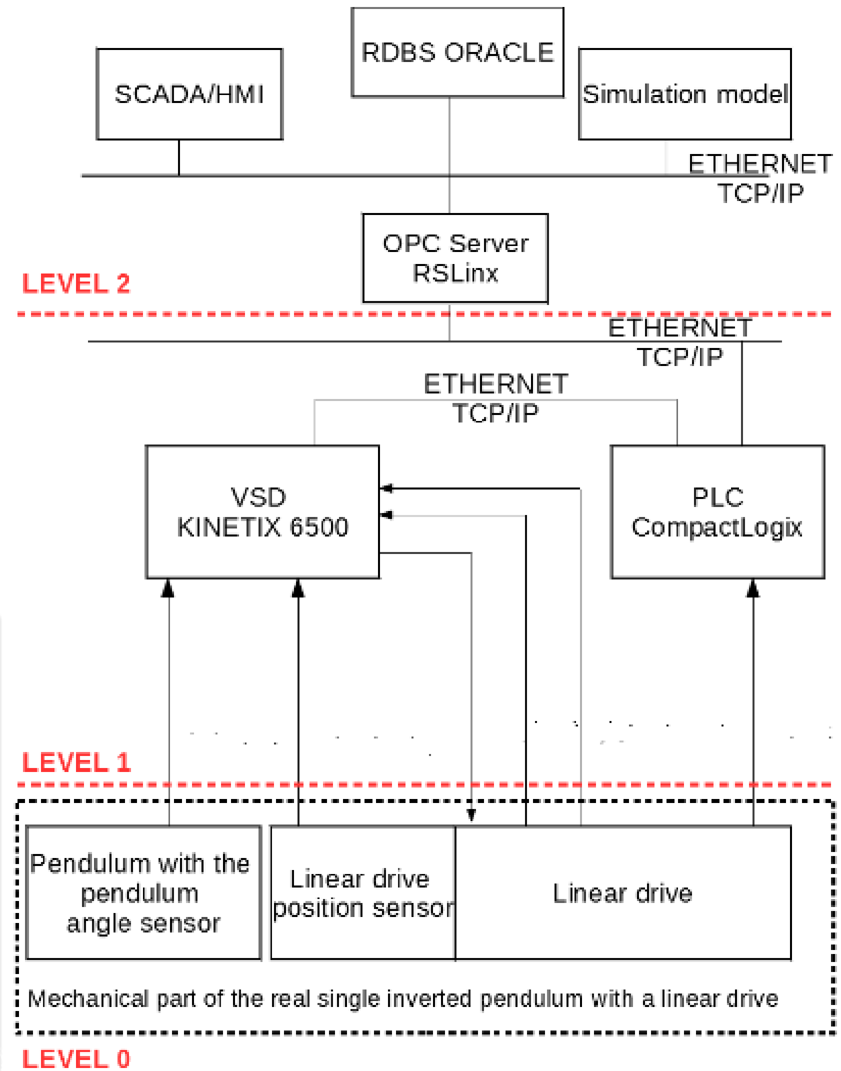
Implementation Into DCS, Identification and Control

Distributed Control System (DCS) Implemented at the DCAI (a Pyramidal Model of a Cyber-physical System)



IMPLEMENTATION OF THE *SINGLE INVERTED PENDULUM LABORATORY MODEL* WITH LSM INTO THE DCS





DEVELOPMENT OF *SIMULATION MODEL LEVEL* FOR THE *SINGLE INVERTED PENDULUM LABORATORY MODEL WITH LSM*

A. System Identification

- Derivation of a mathematical model of a single inverted pendulum on a cart with an attached weight
- Modification of the derived model to include the LSM
- Identification of parameters of the LSM model
- Identification of damping
- Verification against the laboratory model

B. Hybrid Control Design

- Linear approximation
- Design of stabilizing control law – LQR
- Design of swing-up control law – energy shaping
- Verification against the simulation model
- Verification against the laboratory model

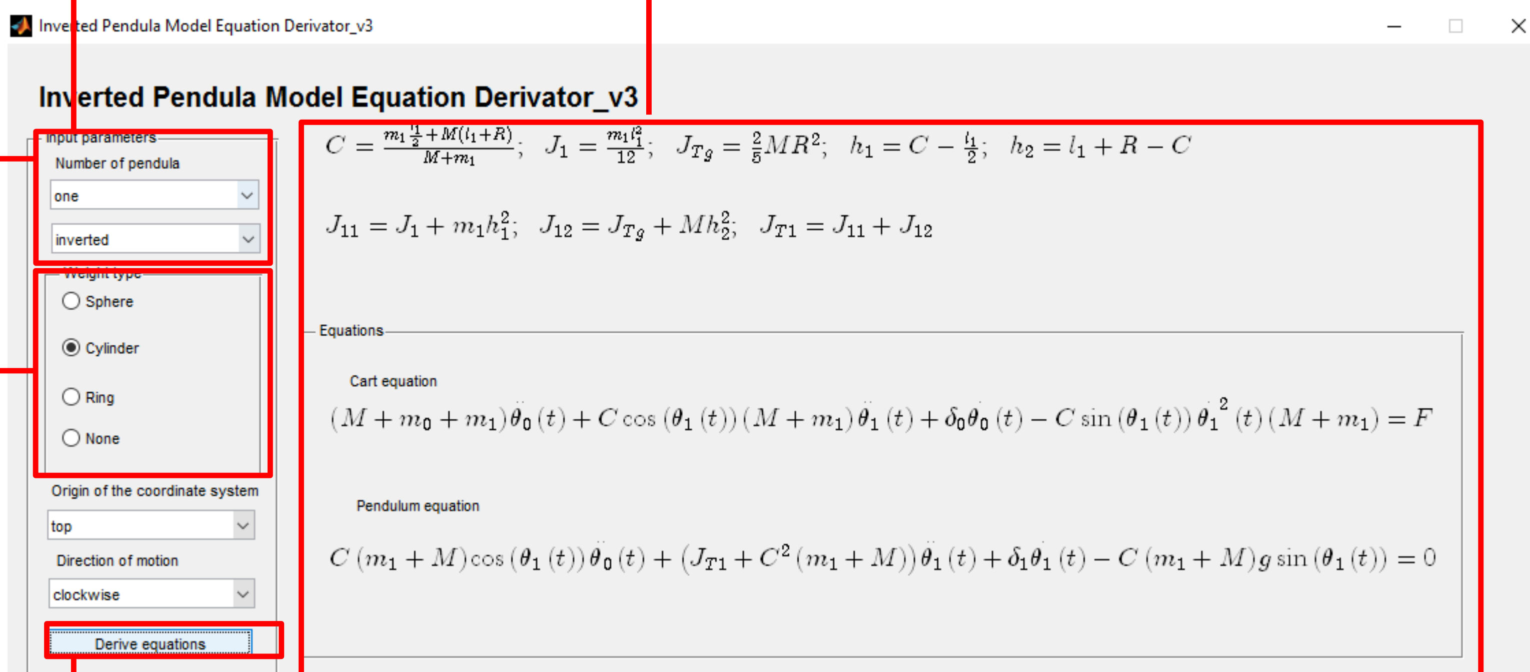
A1. Generating motion equations using the *Inverted Pendula Model Equation Derivator_v3*

system type: classical
number of pendulum links: 1

generated equations of motion

attached weight type: sphere

reference pendulum position: top
direction of pendulum rotation: clockwise



initiates mathematical model derivation

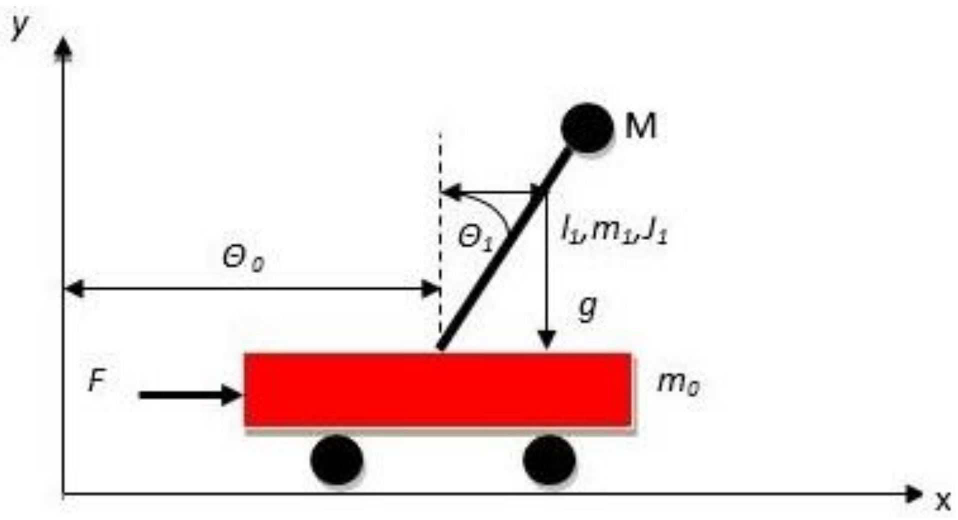
A1. Mathematical Model of a Single Inverted Pendulum on a Cart With an Attached Weight and force input

CART EQUATION

$$(M + m_0 + m_1)\ddot{\theta}_0(t) + C\cos(\theta_1(t))\,(M + m_1)\ddot{\theta}_1(t) +$$
$$+\delta_0\dot{\theta}_0(t) - C\sin(\theta_1(t))\,\dot{\theta}_1^2(t)(M + m_1) = F(t)$$

PENDULUM EQUATION

$$C(m_1 + M)\cos(\theta_1(t))\,\ddot{\theta}_0(t) + J_1\ddot{\theta}_1(t) + \delta_1\dot{\theta}_1(t) - C(m_1 + M)g\sin(\theta_1(t)) = 0$$



DISTANCE BETWEEN PIVOT POINT – CoG

$$C = \frac{m_1\frac{l_1}{2} + M\,(l_1 + R)}{M + m_1}$$

A2. Replacement of the cart equation by *velocity control loop* describing the motor-cart subsystem

SINGLE INVERTED PENDULUM FORCE MODEL – CART EQUATION

$$(M+m_0+m_1)\ddot{\theta}_0(t)+C\cos\left(\theta_1(t)\right)(M+m_1)\ddot{\theta}_1(t)+ \\ +\delta_0\dot{\theta}_0(t)-C\sin\left(\theta_1(t)\right)\dot{\theta}_1^2(t)(M+m_1)=F(t)$$

LSM VELOCITY CONTROLLER INCLUDED INTO THE VELOCITY CONTROL LOOP

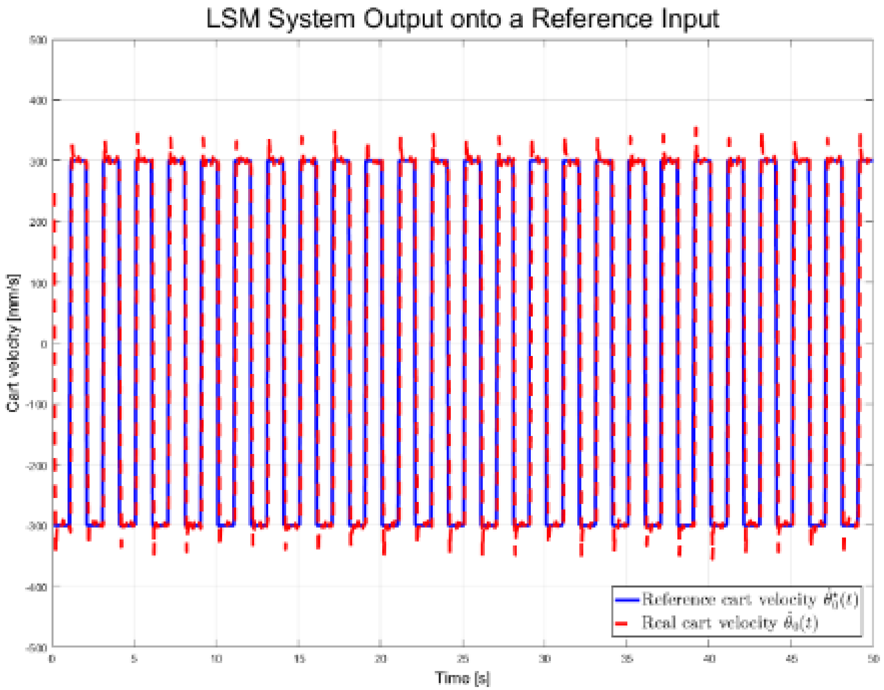
$$\dot{\theta}_0(t)=\dot{\theta}_0^*(t),\quad \ddot{\theta}_0(t)=\ddot{\theta}_0^*(t) \longrightarrow \frac{s\Theta_0(s)}{s\Theta_0^*(s)}=\frac{p_0}{s^2+q_1 s+q_0},$$

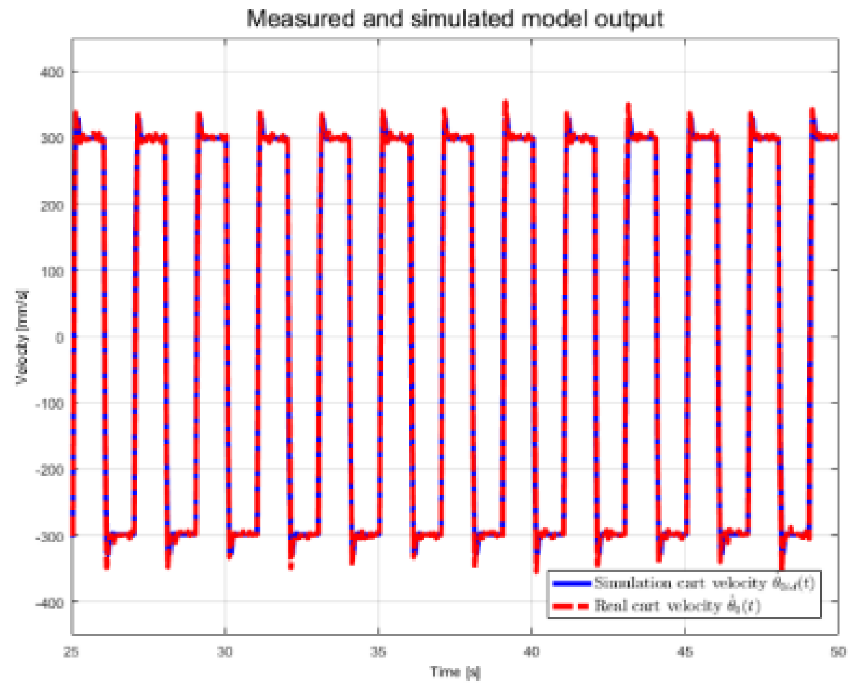
$$\mathcal{L}\left(\dot{\theta}_1(t)\right)=s\Theta_0(s)$$

STATE-SPACE REPRESENTATION OF THE MOTOR-CART SUBSYSTEM WITH VELOCITY CONTROL

$$\frac{d}{dt}\begin{bmatrix}\theta_0(t)\\ \dot{\theta}_0(t)\\ \ddot{\theta}_0(t)\end{bmatrix}=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 0 & -q_0 & -q_1\end{bmatrix}\begin{bmatrix}\theta_0(t)\\ \dot{\theta}_0(t)\\ \ddot{\theta}_0(t)\end{bmatrix}+\begin{bmatrix}0\\ 0\\ p_0\end{bmatrix}\dot{\theta}_0^*(t)$$

A3. Experimental Identification of Linear Drive Velocity Loop Parameters





Reference velocity	$\dot{\theta}_0^*(t)$	***Identified model output***	$\dot{\theta}_{0id}(t)$
Actual velocity	$\dot{\theta}_0(t)$	***Laboratory model output***	$\dot{\theta}_0(t)$

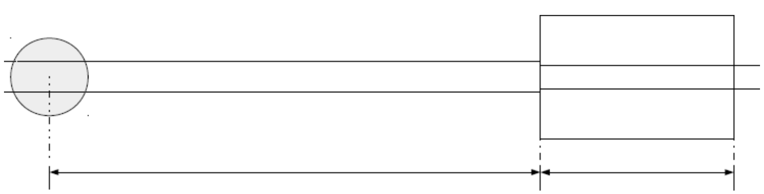
A4. Identification Of The Laboratory System
Damping Coefficient

Damped pendulum equation

$$\ddot{\theta}_1(t) + 2b\dot{\theta}_1(t) + \omega_0^2 \sin(\theta_1) = 0$$

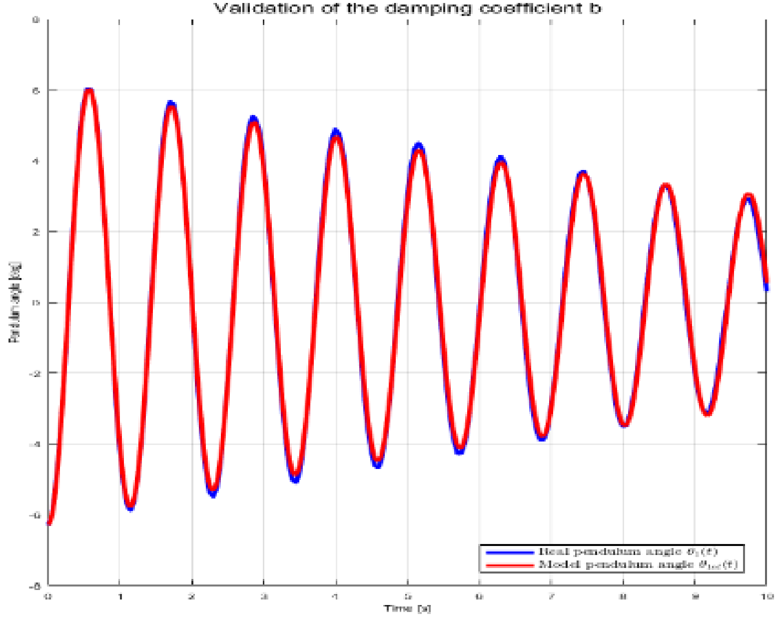
Frequency

$$\omega_0 = \sqrt{\frac{mgC}{J_P}}$$

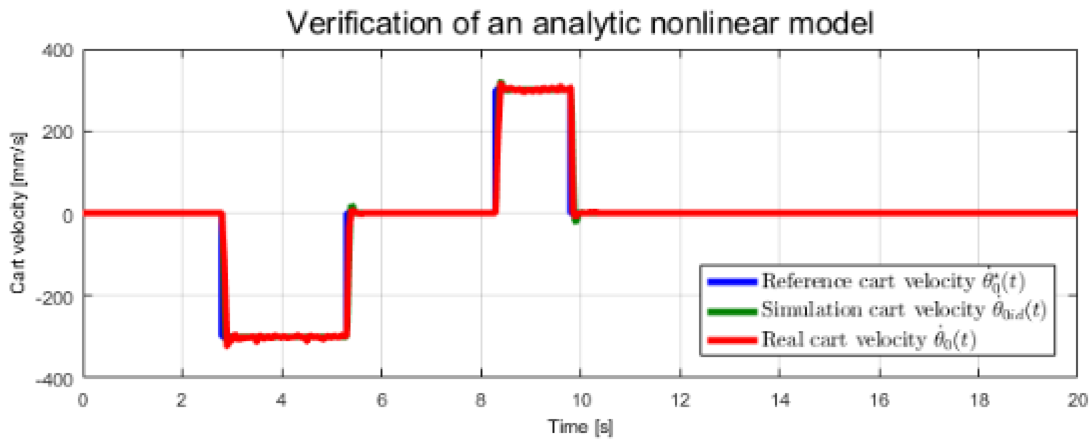


Damping coefficient

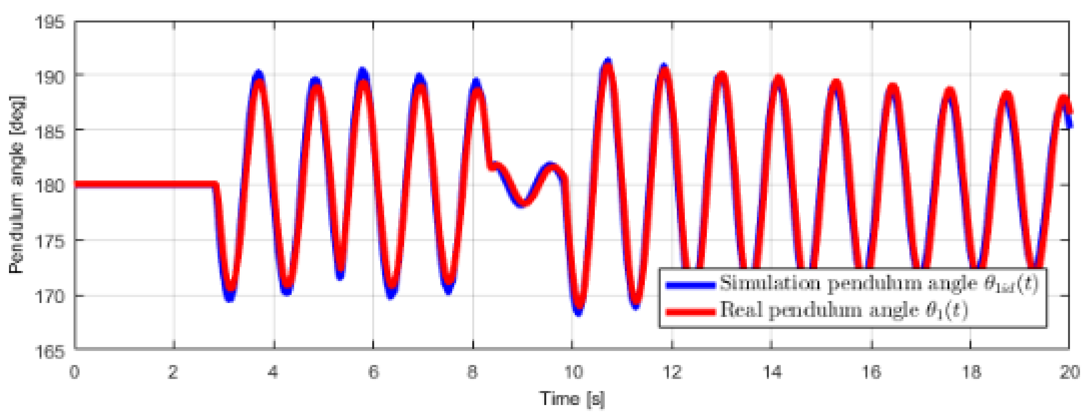
$$b = \frac{1}{nT} \ln\left(\frac{A(t)}{A(t+nT)}\right)$$



A5. Validation of the Final Simulation Model against the Laboratory Model With LSM



CART VELOCITY TIME BEHAVIOR



PENDULUM ANGLE TIME BEHAVIOR

B1. Linear approximation of the modified state-space model

State-space vector

$$\boldsymbol{x} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 \end{bmatrix}^T = \begin{bmatrix} \theta_1(t) & \dot{\theta}_1(t) & \theta_0(t) & \dot{\theta}_0(t) & \ddot{\theta}_0(t) \end{bmatrix}$$

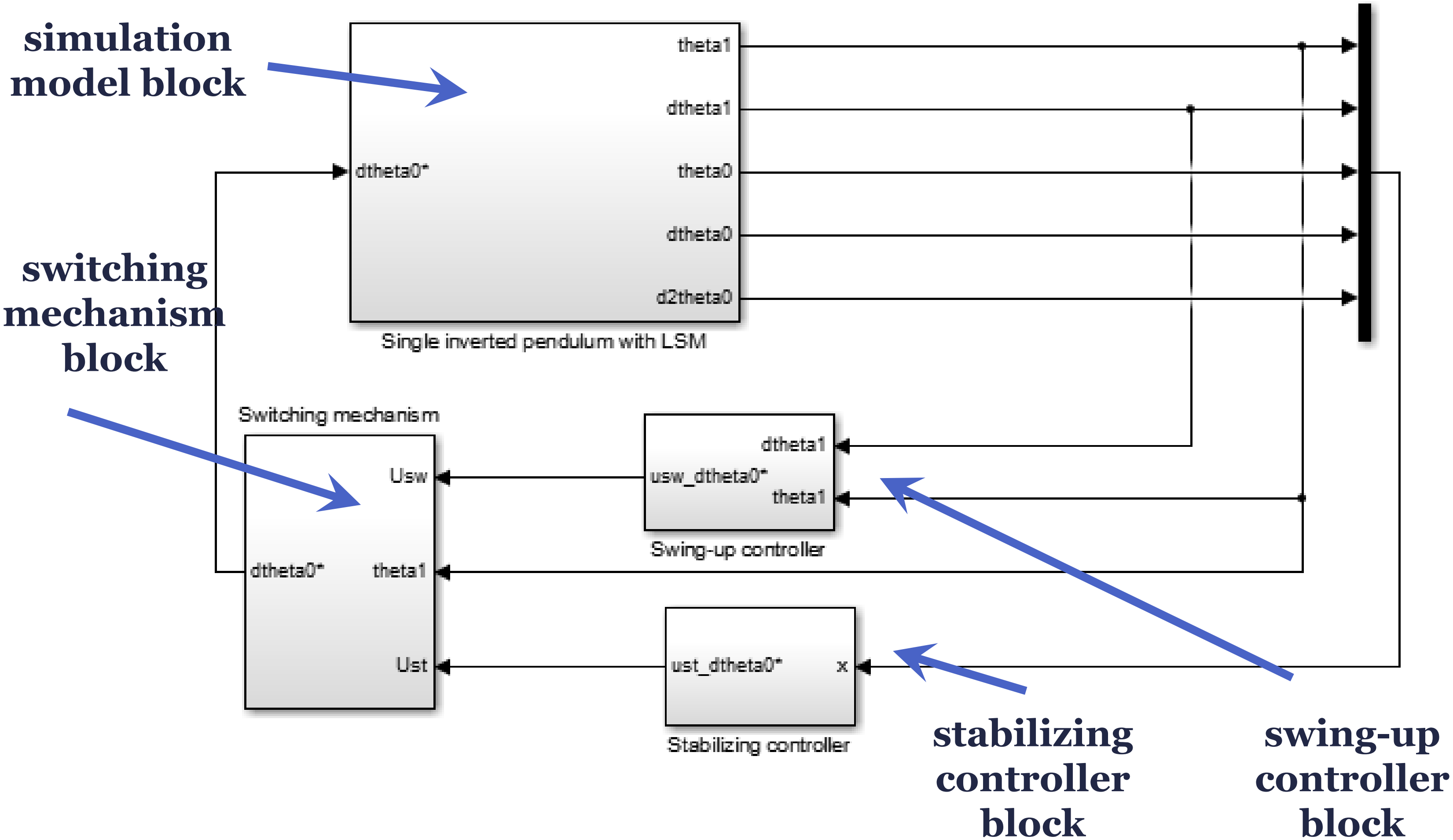
Linear state-space model around the unstable equilibrium

$$\dot{\boldsymbol{x}}(t) = \underbrace{\underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 & a_{25} \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & a_{54} & a_{55} \end{bmatrix}}_{A} \boldsymbol{x}(t) + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ b_5 \end{bmatrix}}_{B}}\, u(t)$$

[k,s,e]=lqr(A,B,Q,R)

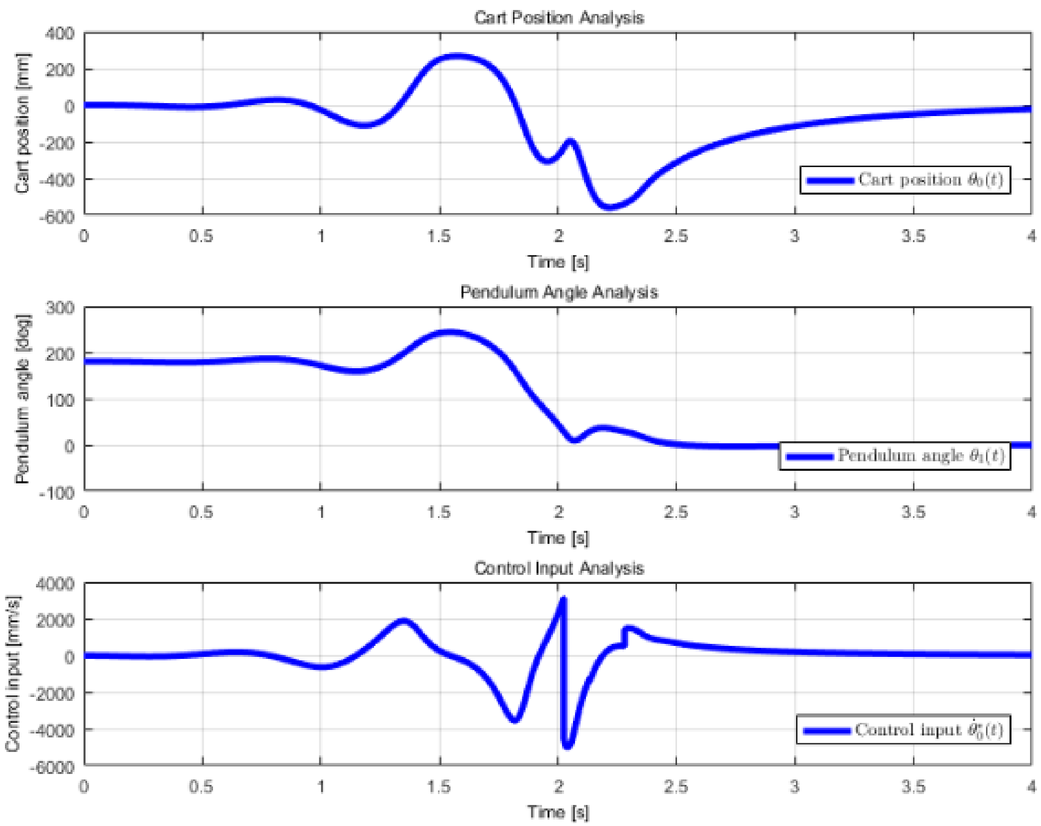
B2.+B3. Hybrid Control Structure for the Single Inverted Pendulum with velocity control

VERIFICATION OF PROPOSED CONTROL DESIGN USING MODIFIED IPMaC BLOCKS



B4. Hybrid Control Design (Swing-up and Stabilization) – Verification On The Simulation Model

SWING-UP BY ENERGY-BASED METHODS, STABILIZATION BY LQR



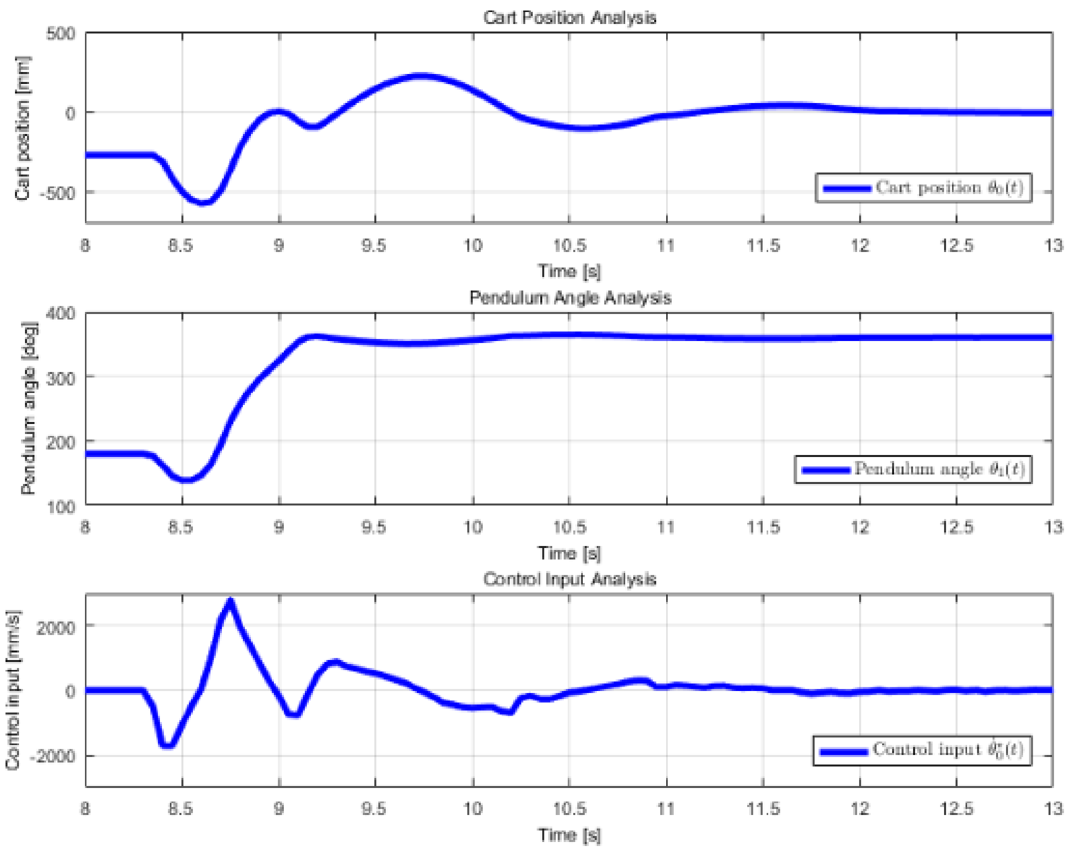
Cart position time behavior

Pendulum angle time behavior

Control input

B5. Hybrid Control Design (Swing-up and Stabilization) – Verification On The Laboratory Model

SWING-UP BY ENERGY-BASED METHODS, STABILIZATION BY LQR

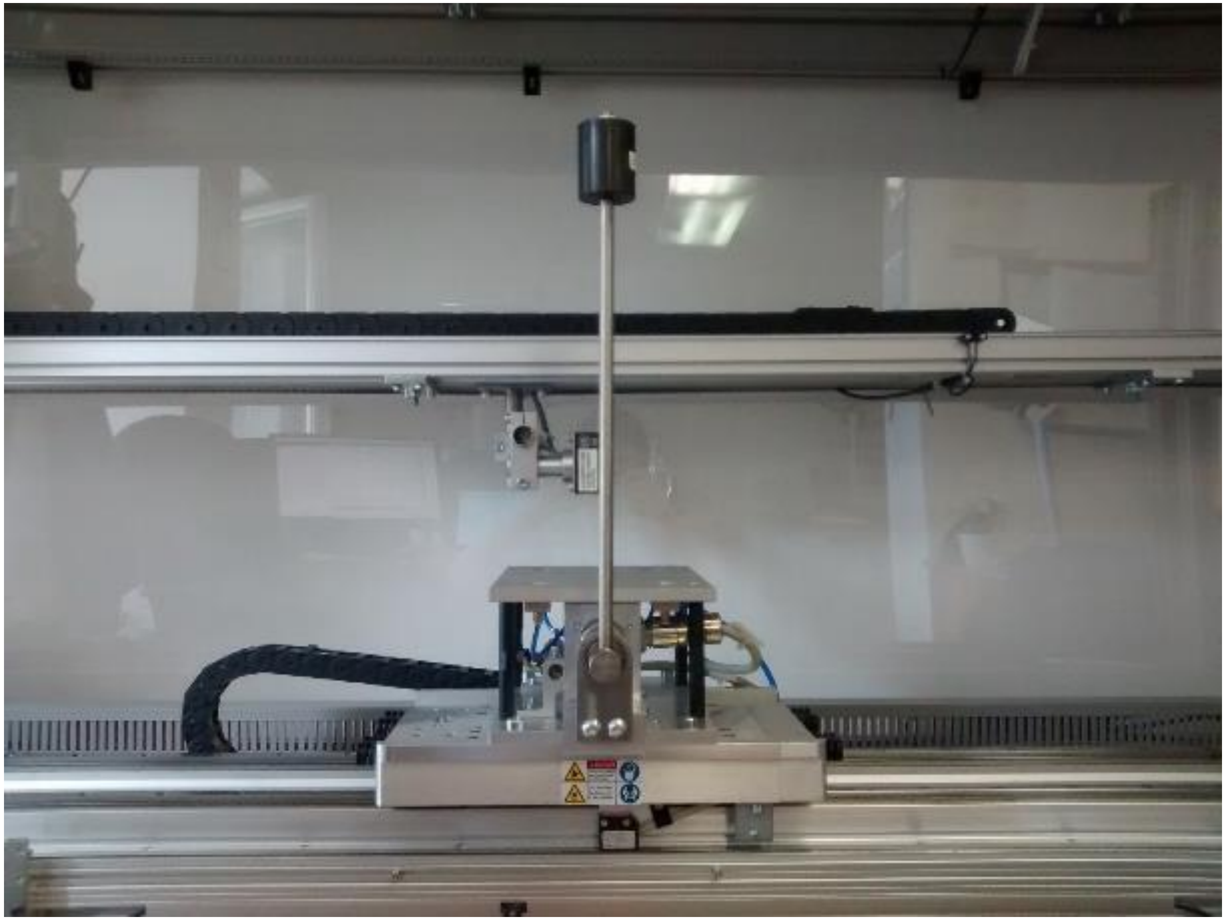


Cart position time behavior

Pendulum angle time behavior

Control input

IDENTIFICATION AND CONTROL OF THE SINGLE INVERTED PENDULUM LABORATORY MODEL – FINAL RESULTS



https://www.youtube.com/watch?v=smrj36WfveQ

SUMMARY

$$
\begin{aligned}
&(M+m_0+m_1)\ddot{\theta}_0(t) + C\cos\left(\theta_1(t)\right)(M+m_1)\ddot{\theta}_1(t) \\
&+\delta_0\dot{\theta}_0(t) - C\sin\left(\theta_1(t)\right)\dot{\theta}_1^2(t)(M+m_1) = F(t)
\end{aligned}
$$

$$
\begin{aligned}
&C(m_1+M)\cos\left(\theta_1(t)\right)\ddot{\theta}_0(t) + J_1\ddot{\theta}_1(t) + \\
&+\delta_1\dot{\theta}_1(t) - C(m_1+M)g\sin\left(\theta_1(t)\right) = 0
\end{aligned}
$$