Motivation Why to Control Feedback History

Jirka Roubal

roubalj@control.felk.cvut.cz

⟨http://support.dce.felk.cvut.cz/pub/roubalj/⟩

Department of Control Engineering, Faculty of Electrical Engineering

Czech Technical University in Prague

⟨www.cvut.cz⟩

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Contents

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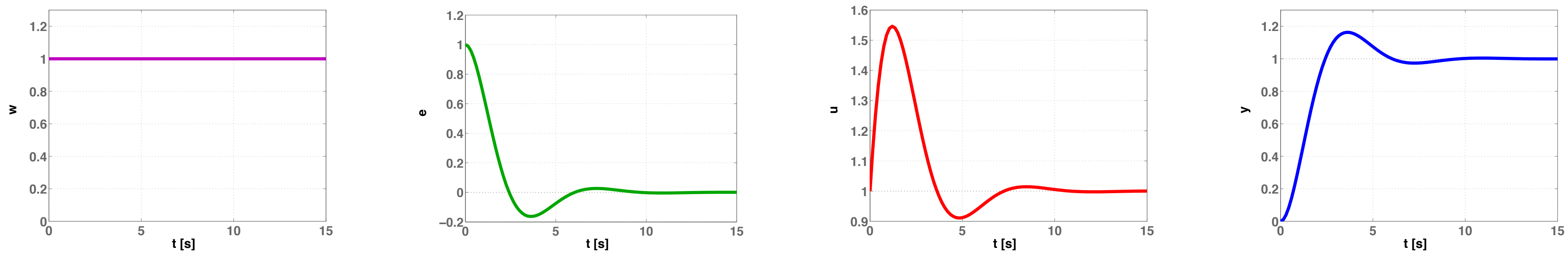
1 Basic Idea of Regulation $=$ FEEDBACK



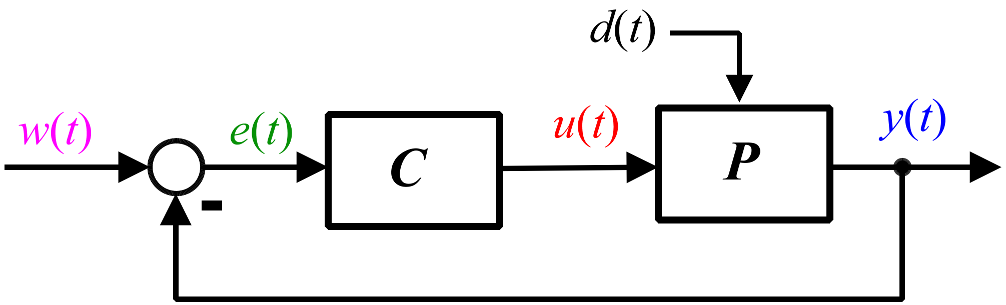
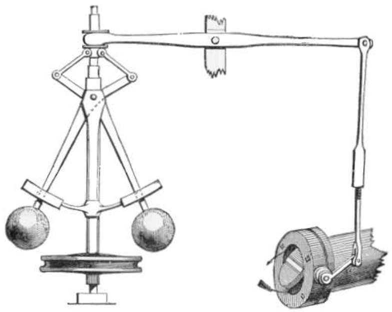
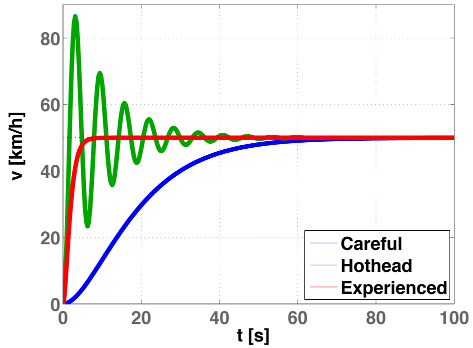
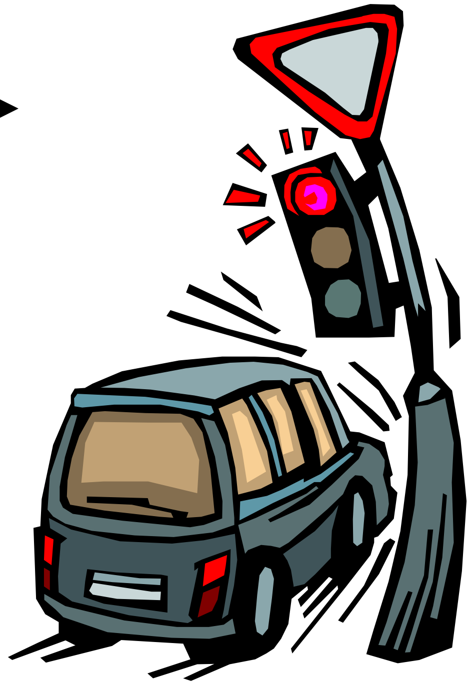


Figure 1: Control feedback loop







(*Wikipedia – Open encyclopedia* [online] 2008)

2 A Brief History of the Control

- The Origins of Feedback Control (MAYR, O. 1970)
- industrial revolution in Europe in the 17th century
- Watt controller of the steam engine
- (MAXWELL, J. C. 1868)
  - the first mathematical article about the feedback
  - why? ... stability problem
- to the end of 19th century – primary period
  - big bloom – 1. and 2. World War
- till 1960 – classical period
- from 1960 – modern period

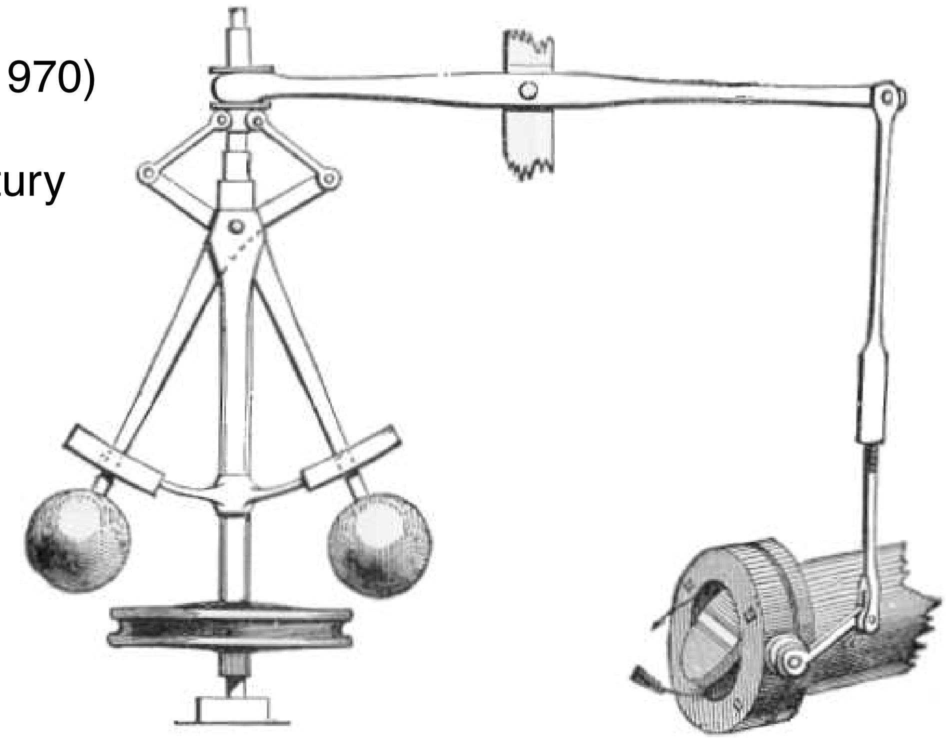


Figure 2: Watt controller of the steam engine

(*Wikipedia – Open encyclopedia* [online] 2008)

But the feedback had been here much before ...

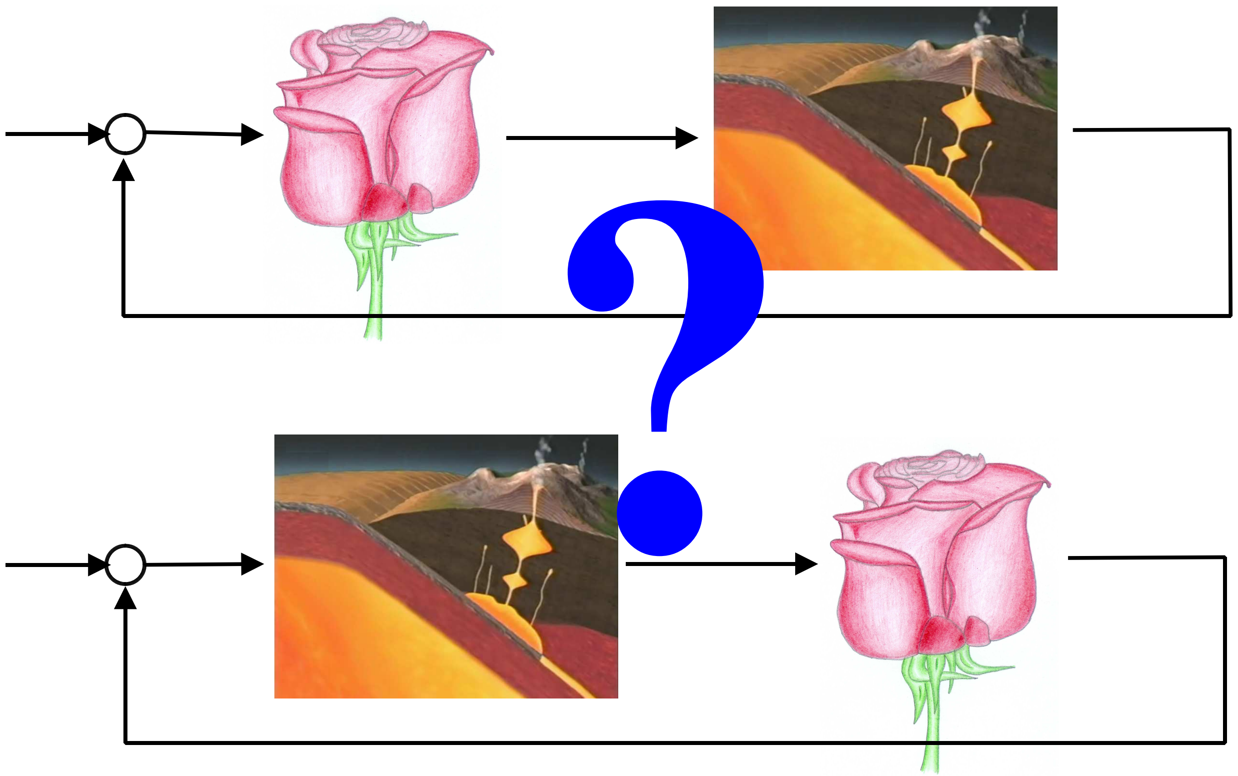


Figure 3: Feedback: life on the Earth – carbon dioxide quantity in the atmosphere

(BBC The Learning Channel 1998)

... and feedback is still here now!!!

- feedback teacher – student
- the quality of the result (your skills) depends especially on the feedback
- which is the most important part of the feedback?

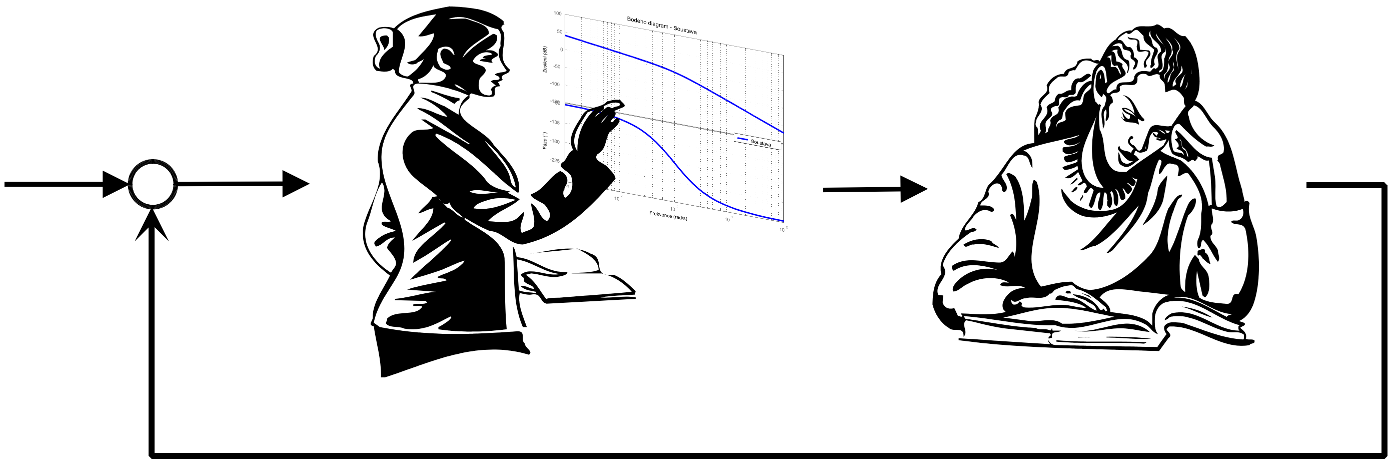


Figure 4: Feedback: teacher – student

- ⟹ if you do not ask, you will learn nothing!!!

3 Control System Design – Example: Coupled Tanks

1. Determination of system inputs and system outputs
2. Finding the mathematical model of the dynamic (and static) system behaviour
3. System identification (determination of the system parameters)
4. Verification of the mathematical model
5. Linearization of the mathematical model and verification of the linearized model
6. Design of the controller based on the linearized model
7. Verification of the controller with the linearized model by the simulation in a computer
8. Verification of the controller with the nonlinear model by the simulation in a computer
9. Application of the controller to the real system

3.1 Example – Controller design for the coupled tanks system

This is a model of the coupled tanks, which consists of a rotary-pump, two tanks (left and right), inflow and outflow pipes and transfer and output valves. The left tank is filled with the fluid by the rotary-pump. The fluid can drain away back through the rotary-pump and drains away to the right tank through the transfer valve and from the right tank drains away through the output valve. The system input is the voltage of the motor of the rotary-pump and the system outputs are the levels of the fluid in the tanks. The transfer valve is a proportional valve and the output valve is a on/off valve.

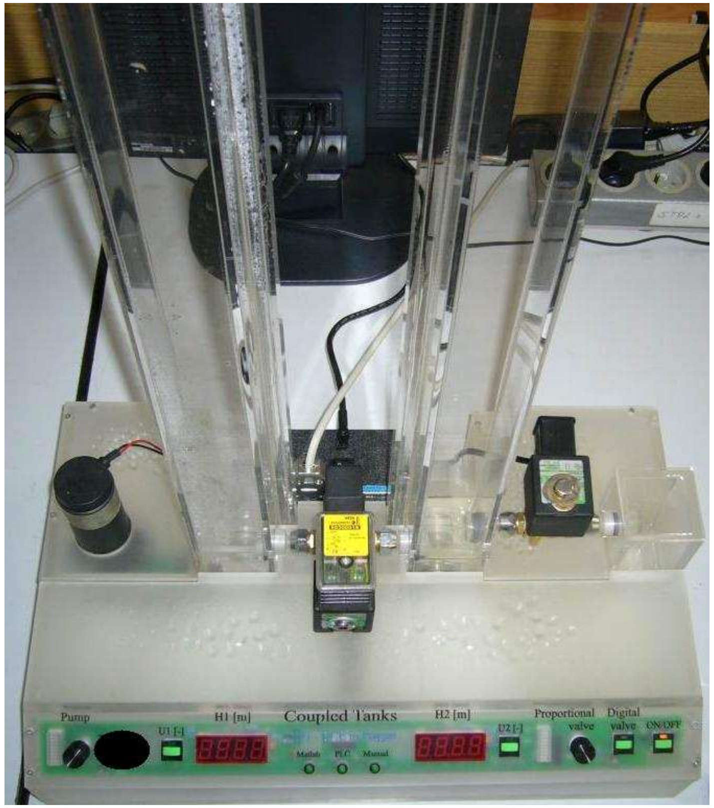


Figure 5: Coupled tanks system

3.1.1 Determination of system inputs and system outputs

- system inputs
  - voltage of the rotary-pump $u(t)$ (action variable)
  - transfer valve opening (disturbance, parameter)
  - output valve opening (disturbance, parameter)
- system output
  - level in the left tank $y_1(t) = h_1(t)$
  - level in the right tank $y_2(t) = h_2(t)$

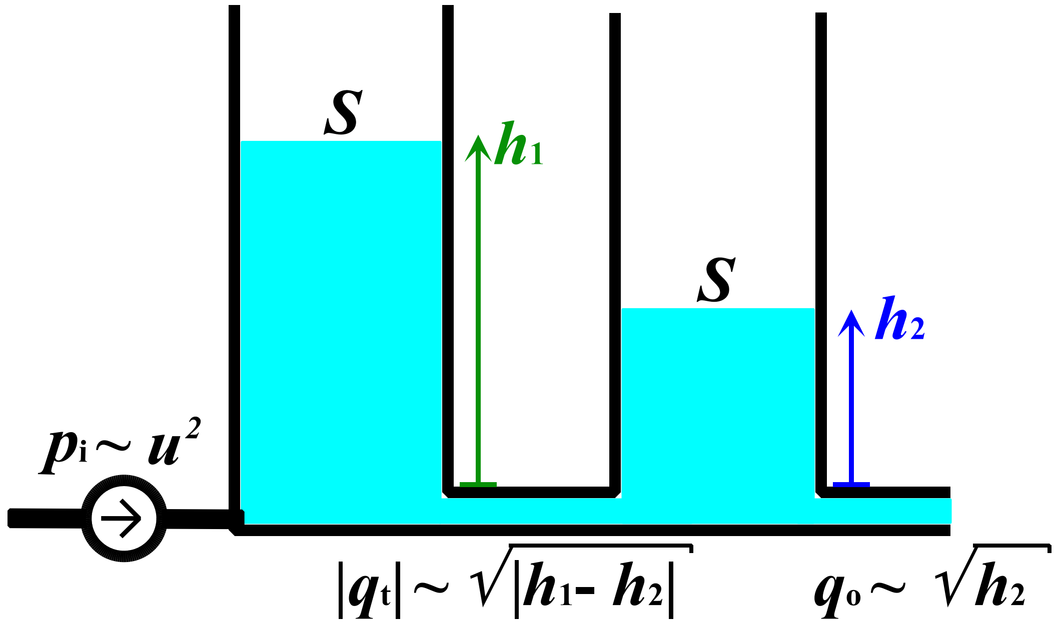


Figure 6: Coupled tanks with a rotary-pump

3.1.2 Finding the mathematical model of the dynamic (and static) system behaviour

$$p_\mathrm{i}(t) = k_\mathrm{i}\, u^2(t)$$

$$\frac{dh_1(t)}{dt} = -\frac{S_\mathrm{t}}{S}\sqrt{2g}\;\mathrm{sign}\big(h_1(t)-h_2(t)\big)\sqrt{|h_1(t)-h_2(t)|} + \\ +\frac{S_\mathrm{i}}{S}\;\mathrm{sign}\left(\frac{p_\mathrm{i}(t)}{\rho}-gh_1(t)\right)\sqrt{2\left|\frac{p_\mathrm{i}(t)}{\rho}-gh_1(t)\right|}$$

$$\frac{dh_2(t)}{dt} = +\frac{S_\mathrm{t}}{S}\sqrt{2g}\;\mathrm{sign}\big(h_1(t)-h_2(t)\big)\sqrt{|h_1(t)-h_2(t)|} - \\ -\frac{S_\mathrm{o}}{S}\sqrt{2g}\,\sqrt{h_2(t)}$$

- more detailed derivation of this model can be found in (ROUBAL, J., HUŠEK, P. & SPOL. 200x)
- p_i is the pressure that is caused by the rotary-pump
- k_i is the constant of the rotary-pump
- ρ is density of the fluid, g is the gravitational constant
- S_i, S_t, S_o S, cross section of input, transfer, output valve and cross section of the tanks

3.1.3 System identification (determination of the system parameters)

- static characteristics
  - Rotary-pump
    - $u_{\min} = 0.01$ V corresponding to $h_{1\,\min} = -0.050$ m
    - $u = 0.160$ V when the level in the tank is starting to rise
    - $u = 0.250$ V corresponding to $h_1 = 0.000$ m
    - $u = 0.603$ V corresponding to the maximum level in the left tank
    - $u_{\max} = 1$ V
  - Levels in the tanks
    - $h_{1\,\min} = -0.050$ m $h_{1\,\max} = 0.665$ m
    - $h_{2\,\min} = 0.000$ m $h_{2\,\max} = 0.670$ m

The values on the system board approximately correspond to the values in Matlab (THE MATHWORKS 2008). Only the offsets of the levels in the tanks are compensated such that the zero levels correspond to the positions of the valves (zero values on the system board).

- identification of constant k_i
  - transfer valve is closed
  - output valve can be closed or open
  - for steady state level in the left tank

$$0 = \frac{k_\mathrm{i}\, u^2}{\rho} - g h_1$$

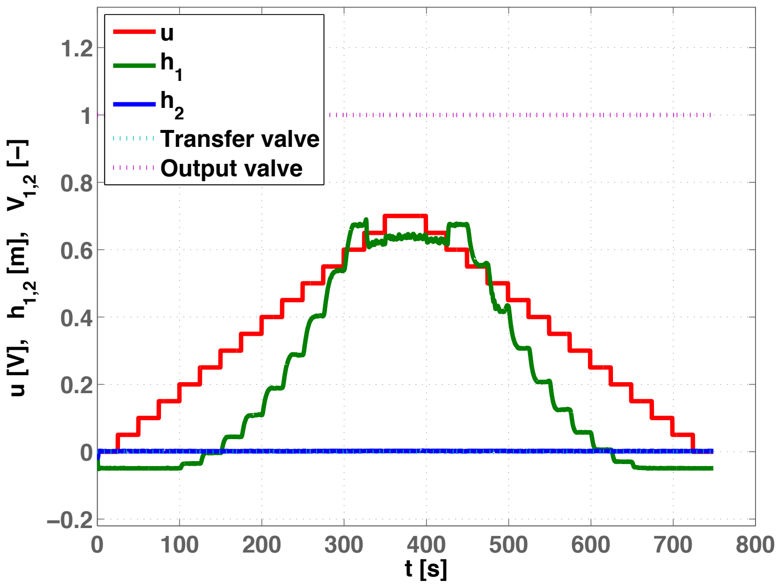


Figure 7: Experiment to obtain constant k_i

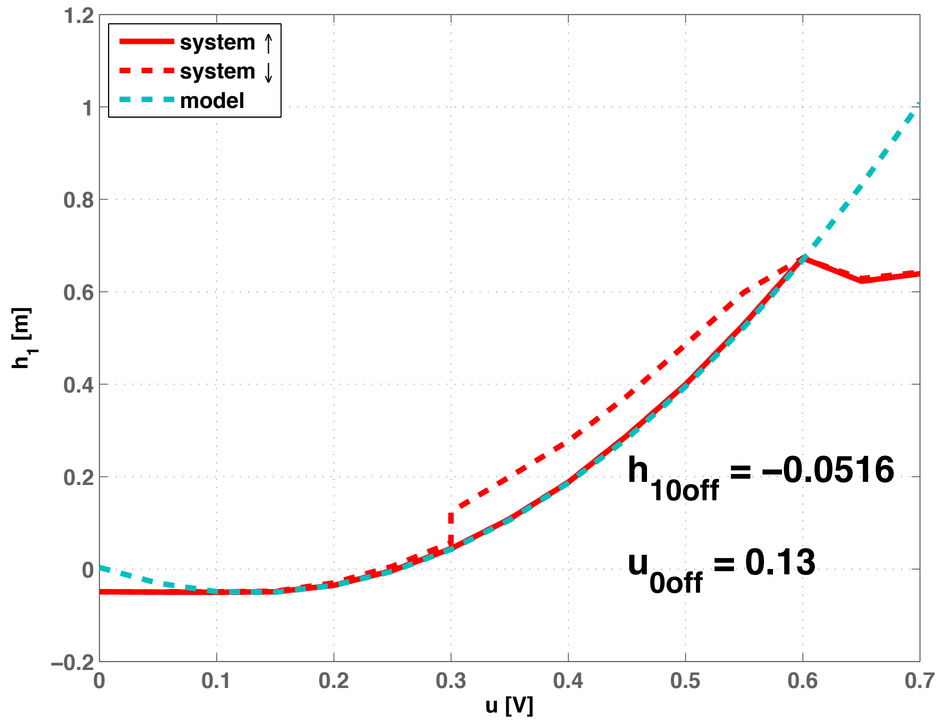


Figure 8: Static characteristic of the rotary pump

from Figure 8 you can see that $\frac{k_\mathrm{i} u^2}{\rho} - g h_1$

$$\Longrightarrow \frac{k_\mathrm{i}\, \left(u(t) - u_{0\mathrm{off}}\right)^2}{\rho} - g \left(h_1(t) - h_{10\mathrm{off}}\right)$$

$$k_\mathrm{i} = 32000\,\mathrm{kg\,m^{-1}\,s^{-2}\,V^{-2}}$$

- $\rho = 1000\,\mathrm{kg\,m^{-3}}$, $g = 9.81\,\mathrm{m\,s^{-2}}$, $S = 50^2\,\mathrm{mm^2}$
- from the experiment from Figure 7, the constant S_i can be obtained

$$\frac{dh_1(t)}{dt} = +\frac{S_\mathrm{i}}{S}\,\mathrm{sign}\left(\frac{k_\mathrm{i}u^2(t)}{\rho} - gh_1(t)\right)\sqrt{2\left|\frac{k_\mathrm{i}u^2(t)}{\rho} - gh_1(t)\right|}$$

$$S_\mathrm{i} = 35\,\mathrm{mm^2}$$

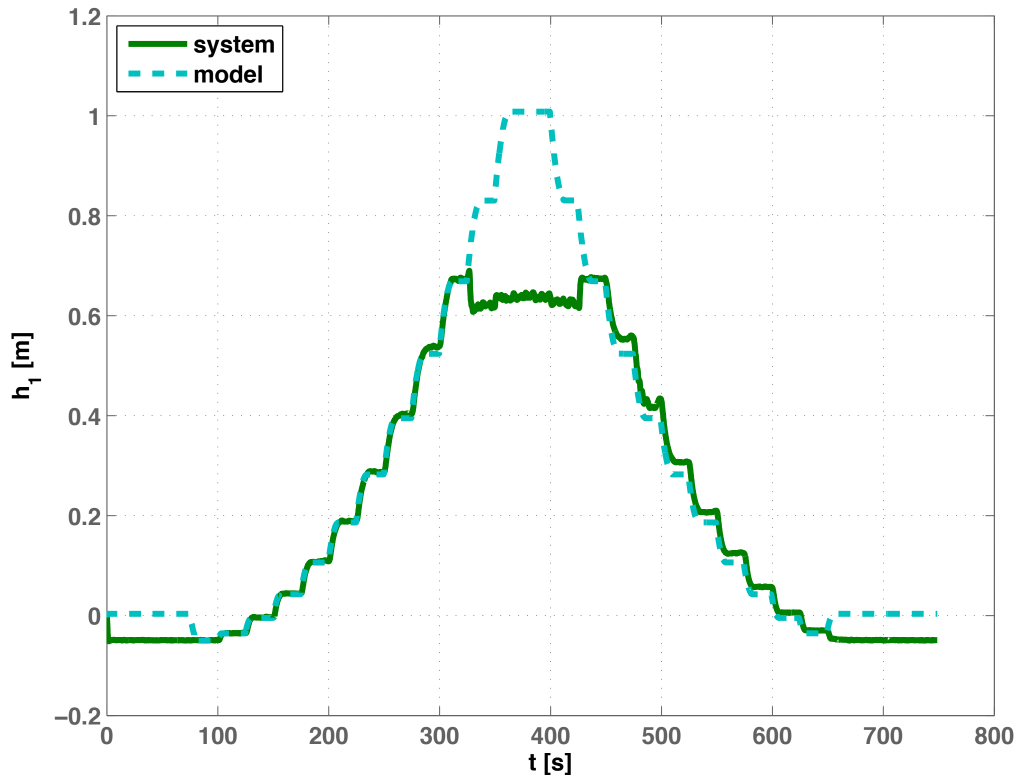


Figure 9: Verification of the constant S_i (the input pipe)

- identification of constant S_t
  - transfer valve is closed to 70 s and then open
  - output valve is closed
  - the mathematical model loses only the last term in the second differential equation
  - from time response after 70 s, constant S_t is obtained

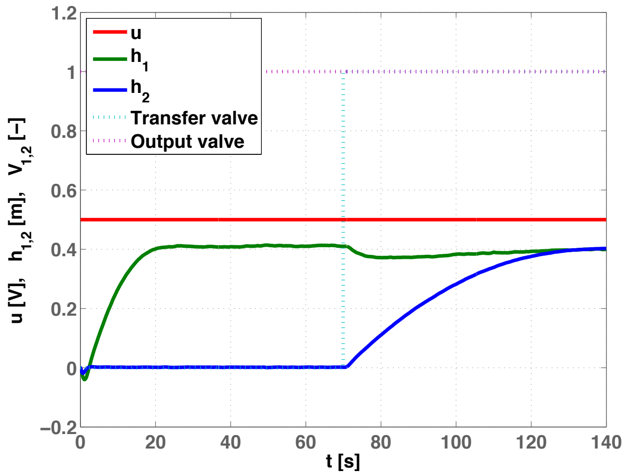


Figure 10: Experiment to obtain constant S_t

$$S_\mathrm{t} = 11.56\,\mathrm{mm}^2$$

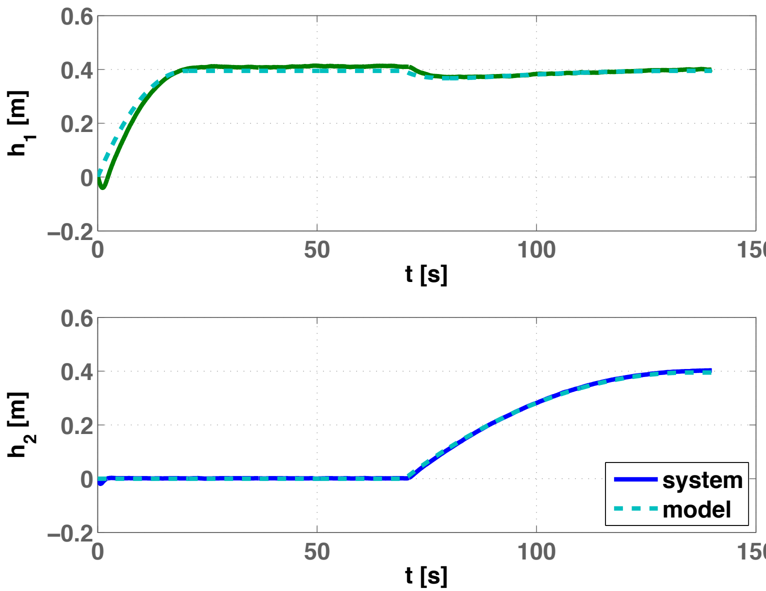


Figure 11: Verification of the constant S_t (the transfer valve)

- identification of constant S_o
  - transfer valve is open to 50 s and then closed
  - output valve is closed to 50 s and then open

$$\frac{dh_2(t)}{dt} = +\frac{S_\mathrm{t}}{S}\,\mathrm{sign}\big(h_1(t)-h_2(t)\big)\sqrt{2g}\sqrt{|h_1(t)-h_2(t)|} - \frac{S_\mathrm{o}}{S}\,\sqrt{2g}\sqrt{h_2(t)}$$

  - from time response after 50 s, constant S_o is obtained

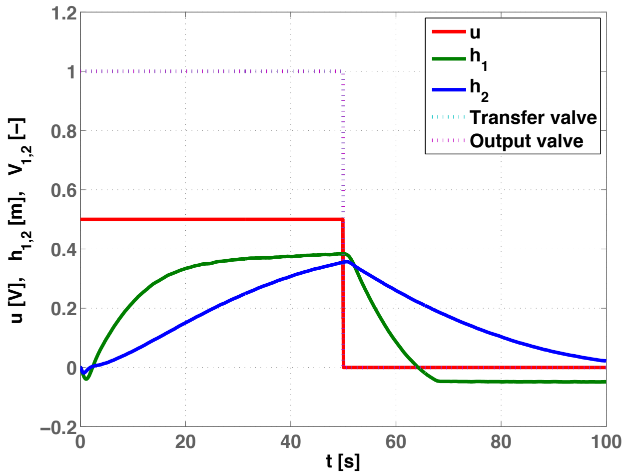


Figure 12: Experiment to obtain constant S_o

$$S_\mathrm{o} = 10.24\,\mathrm{mm}^2$$

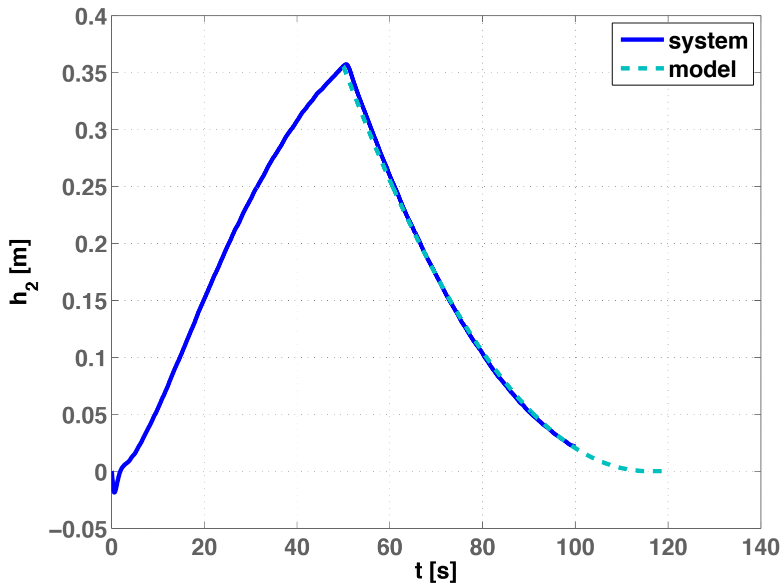


Figure 13: Verification of the constant S_o (the output valve)

3.1.4 Verification of the mathematical model

$$\frac{dh_1(t)}{dt} = -\frac{S_\mathrm{t}}{S}\sqrt{2g}\,\mathrm{sign}\big(h_1(t)-h_2(t)\big)\sqrt{|h_1(t)-h_2(t)|} + \\ +\frac{S_\mathrm{i}}{S}\,\mathrm{sign}\left(\frac{k_\mathrm{i}\big(u(t)-u_{0\mathrm{off}}\big)^2}{\rho} - g\big(h_1(t)-h_{10\mathrm{off}}\big)\right)\sqrt{2\left|\frac{k_\mathrm{i}\big(u(t)-u_{0\mathrm{off}}\big)^2}{\rho} - g\big(h_1(t)-h_{10\mathrm{off}}\big)\right|} \qquad (1)$$

$$\frac{dh_2(t)}{dt} = +\frac{S_\mathrm{t}}{S}\sqrt{2g}\,\mathrm{sign}\big(h_1(t)-h_2(t)\big)\sqrt{|h_1(t)-h_2(t)|} - \frac{S_\mathrm{o}}{S}\sqrt{2g}\,\sqrt{h_2(t)} \qquad (2)$$

$k_\mathrm{i} = 32000\,\mathrm{kg\,m^{-1}\,s^{-2}\,V^{-2}}$ $\quad u_{0\mathrm{off}} = 0.13\,\mathrm{V}$ $\quad h_{10\mathrm{off}} = -0.0537\,\mathrm{m}$

$S = 50^2\,\mathrm{mm^2}$ $\quad S_\mathrm{i} = 35\,\mathrm{mm^2}$ $\quad S_\mathrm{t} = 11.56\,\mathrm{mm^2}$ $\quad S_\mathrm{o} = 10.24\,\mathrm{mm^2}$

$g = 9.81\,\mathrm{m\,s^{-2}}$ $\quad \rho = 1000\,\mathrm{kgm^{-3}}$

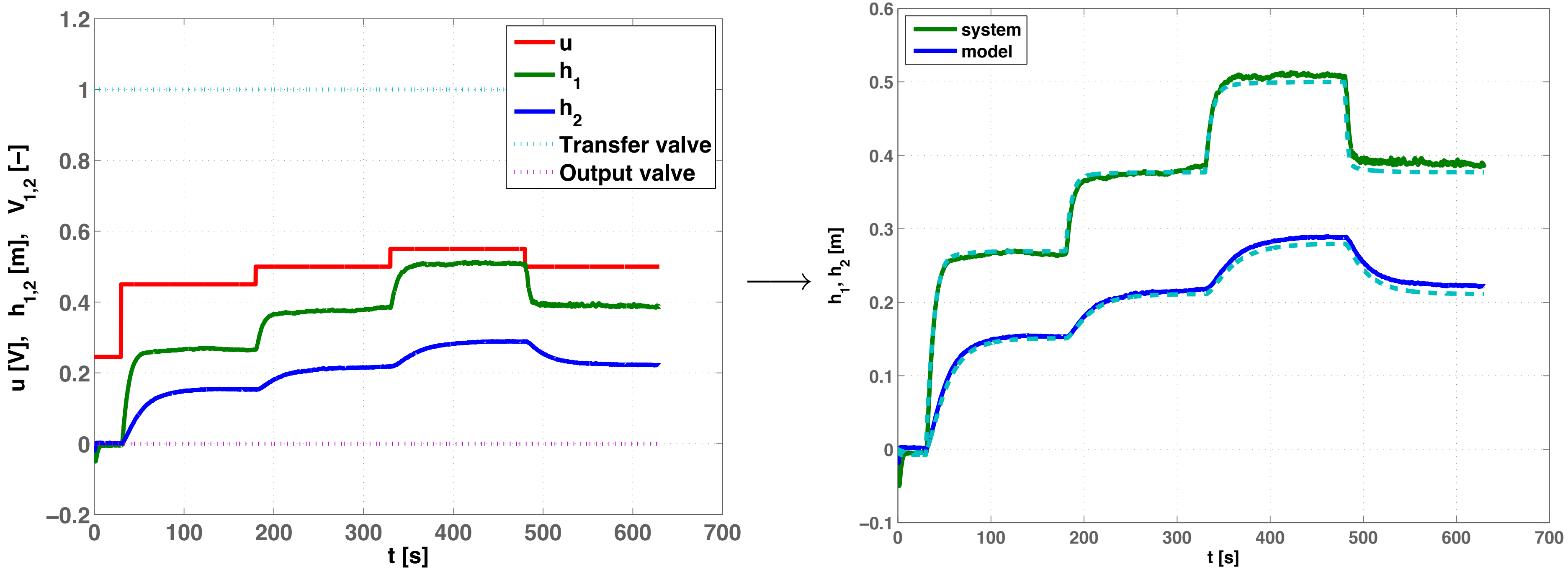


Figure 14: Verification of the whole model

3.1.5 Linearization of the mathematical model and verification of the linearized model

- operating point $u_{0\mathrm{lim}} = 0.45\,\mathrm{V}$ $h_{10\mathrm{lim}} = 0.2695\,\mathrm{m}$ $h_{20\mathrm{lim}} = 0.1504\,\mathrm{m}$

- state space matrices of the linearized model

$$\boldsymbol{A} = \begin{bmatrix} -\frac{S_\mathrm{t}\sqrt{2g}}{2S\sqrt{h_{10\mathrm{lim}}-h_{20\mathrm{lim}}}} - \frac{S_\mathrm{i}\,g}{S\sqrt{2\left(\frac{k_\mathrm{i}(u_{0\mathrm{lim}}-u_{0\mathrm{off}})^2}{\rho}-g(h_{10\mathrm{lim}}-h_{10\mathrm{off}})\right)}} & +\frac{S_\mathrm{t}\sqrt{2g}}{2S\sqrt{h_{10\mathrm{lim}}-h_{20\mathrm{lim}}}} \\ +\frac{S_\mathrm{t}\sqrt{2g}}{2S\sqrt{h_{10\mathrm{lim}}-h_{20\mathrm{lim}}}} & -\frac{S_\mathrm{t}\sqrt{2g}}{2S\sqrt{h_{10\mathrm{lim}}-h_{20\mathrm{lim}}}} - \frac{S_\mathrm{o}\sqrt{2g}}{2S\sqrt{h_{20\mathrm{lim}}}} \end{bmatrix}$$

$$\boldsymbol{B} = \begin{bmatrix} \frac{4S_\mathrm{i}k_\mathrm{i}(u_{0\mathrm{lim}}-u_{0\mathrm{off}})}{2S\rho\sqrt{2\left(\frac{k_\mathrm{i}(u_{0\mathrm{lim}}-u_{0\mathrm{off}})^2}{\rho}-g(h_{10\mathrm{lim}}-h_{10\mathrm{off}})\right)}} \\ 0 \end{bmatrix}$$

$$\boldsymbol{C} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\boldsymbol{D} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

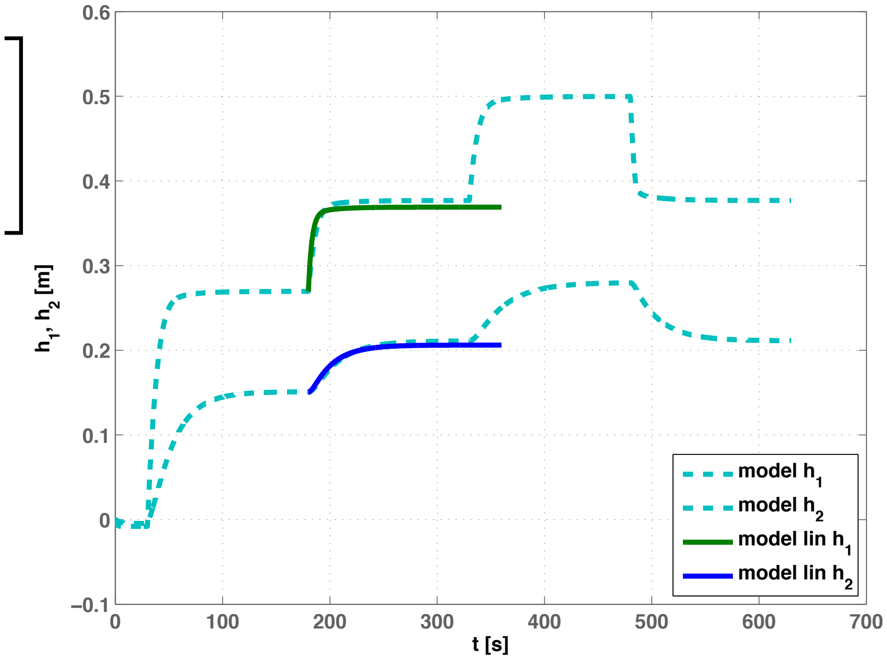


Figure 15: The nonlinear and linearized model comparison

- linearized model is OK

3.1.6 Design of the controller based on the linearized model

- operating point $u_{0\mathrm{lim}} = 0.45\,\mathrm{V}$ $h_{10\mathrm{lim}} = 0.2695\,\mathrm{m}$ $h_{20\mathrm{lim}} = 0.1504\,\mathrm{m}$

$$P_1(s) = \frac{H_1(s)}{U(s)} = \frac{0.1554s + 0.008247}{s^2 + 0.12s + 0.00267} \qquad P_2(s) = \frac{H_2(s)}{U(s)} = \frac{0.004613}{s^2 + 0.12s + 0.00267} \tag{3}$$

- controller design based on model $P_2(s)$ in (3) by Root Locus Method (FRANKLIN, G. F., POWELL, J. D. & EMAMI-NAEINI, A. 2005)

$$C(s) = \frac{85.6436(s + 0.05406)(s + 0.54)}{s(s + 5)} \tag{4}$$

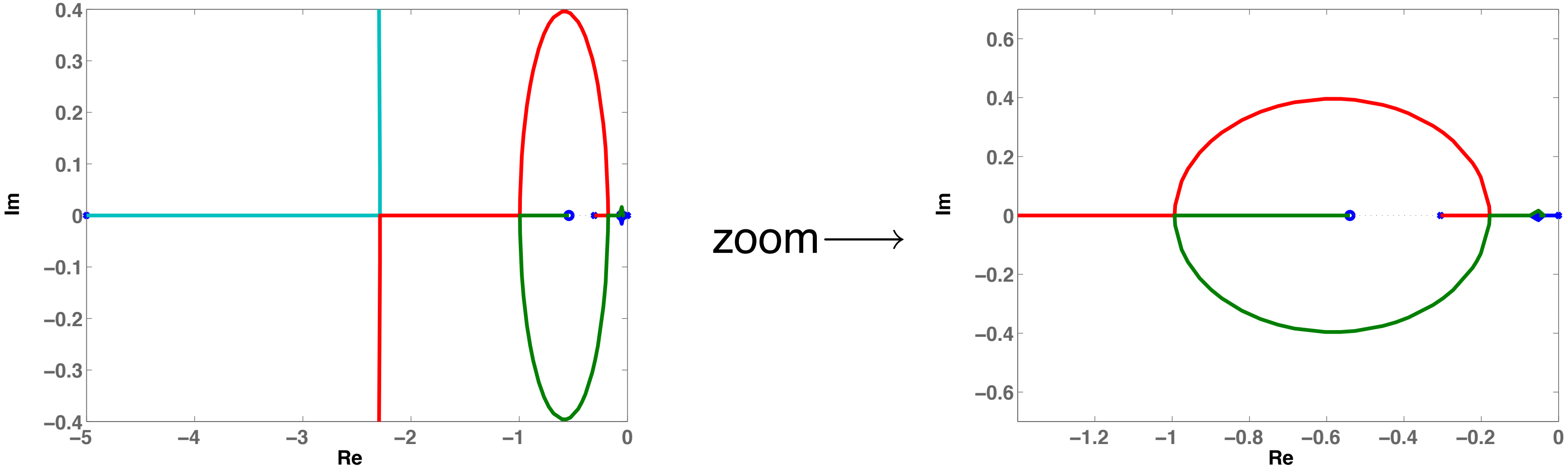


Figure 16: The Root Locus with controller $C(s)$ and model $P_2(s)$ – distribution of the closed loop poles of the feedback loop (just now it is not interesting what the RL Method is)

3.1.7 Verification of the controller with the linearized model (simulation in a computer)

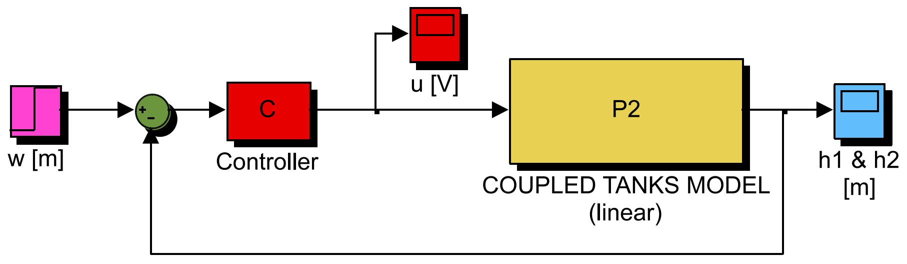


Figure 17: The closed loop with the linearized model

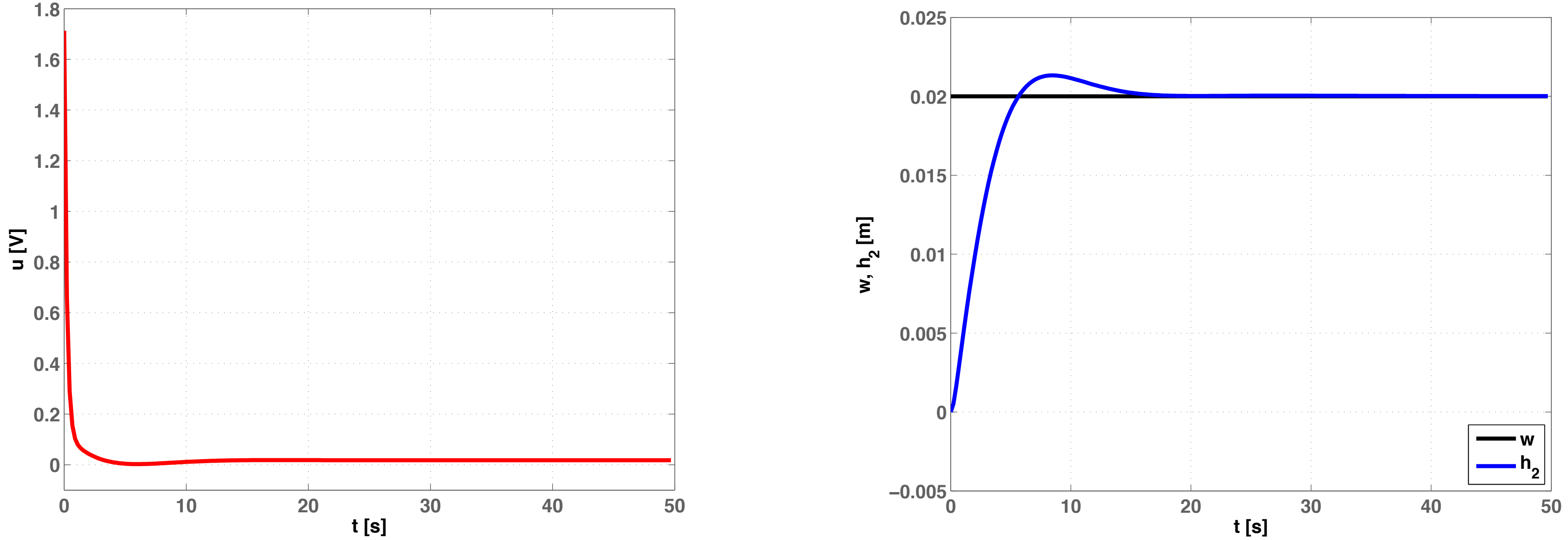


Figure 18: The system input $u(t)$ and the system output $y(t)$ time responses

3.1.8 Verification of the controller with the nonlinear model (simulation in a computer)

- think about $u_{0\mathrm{lin}}$ constant in the closed loop in Figure 19
- do not forget to set h_{10}, h_{20} in the nonlinear model

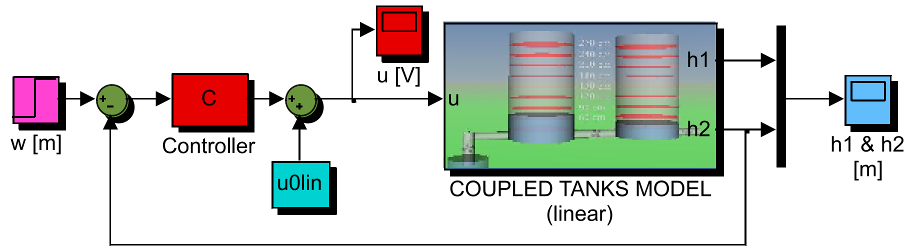
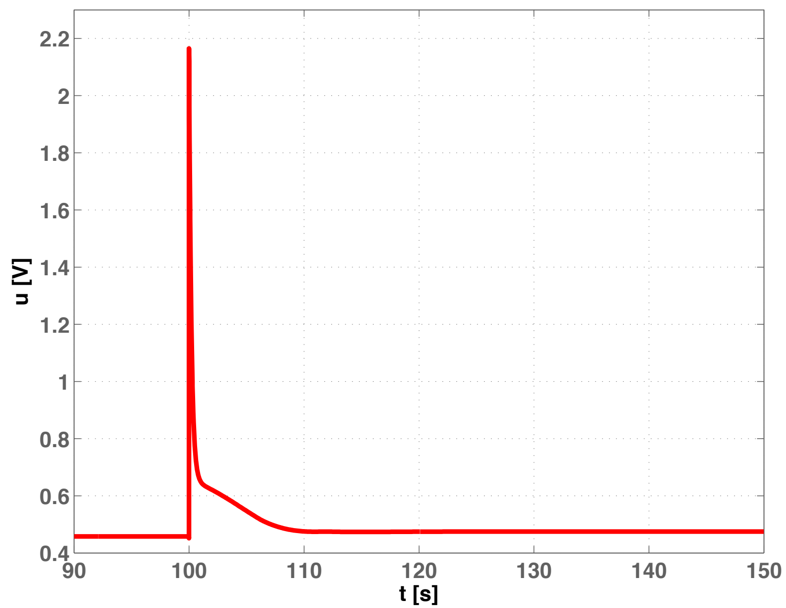


Figure 19: The closed loop with the nonlinear model



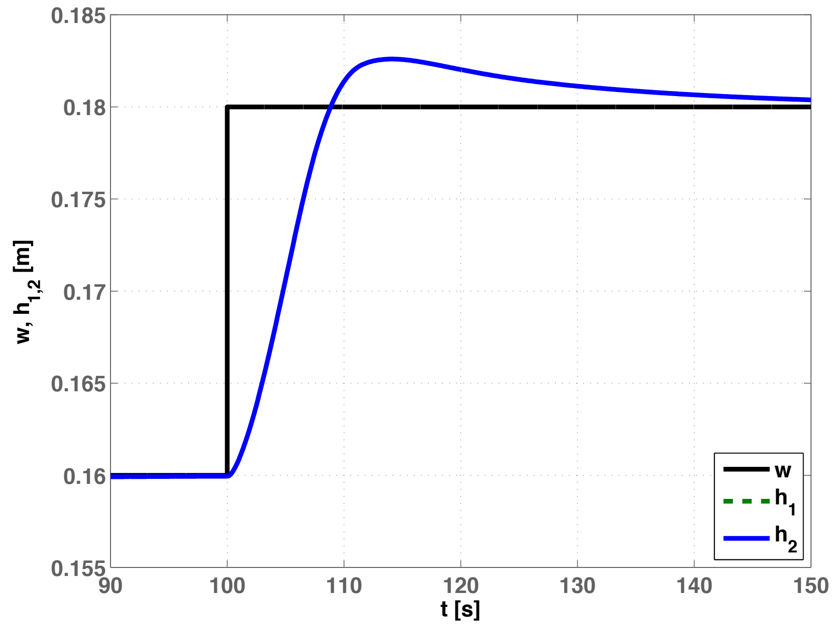


Figure 20: The system input $u(t)$ and the system output $y(t)$ time responses

- we can test our controller from the zero fluid levels
- the closed loop works quite good because the closed loop has a good robustness (ZHOU, K., DOYLE, J. C. & GLOVER, K. 1996) (= the controller is able to control the system which does not correspond to the model exactly)

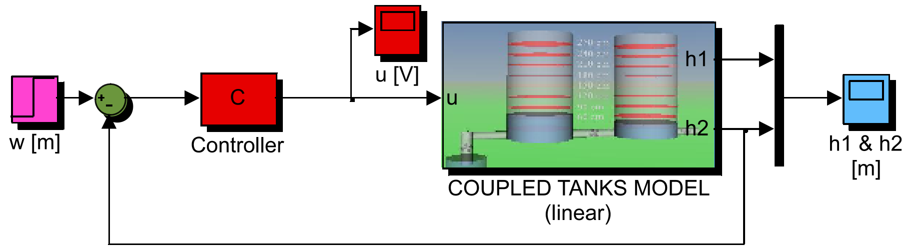


Figure 21: The closed loop with the nonlinear model

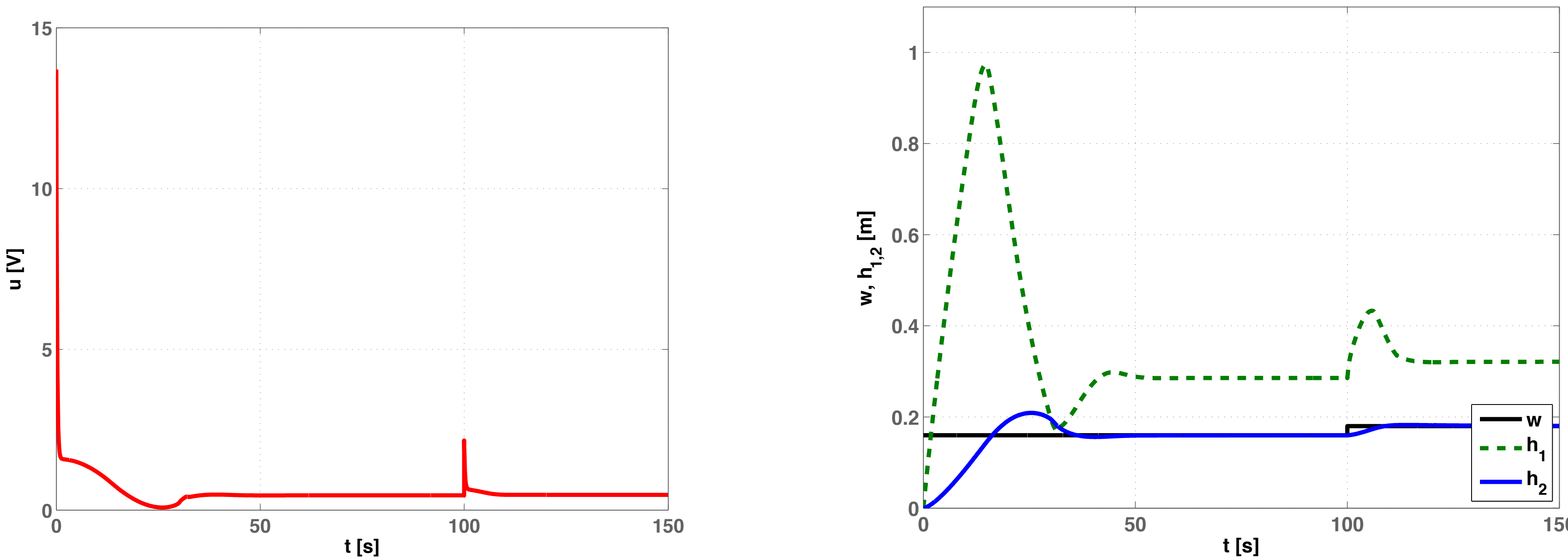


Figure 22: The system input $u(t)$ and the system output $y(t)$ time responses

3.1.9 Application of the controller to the real system

- the simulation before was very good, so we can apply designed controller (4) to the real system, see Figure 23, what is wrong in this scheme;
- the time responses of the system in the closed loop are in Figure 24 and Figure 25
- we should more analyze the model (CHEN, C. T. 1998) and we can design other advance controller (ÅSTRÖM, K. J. & WITTENMARK, B. 1997, HAVLENA, V. & ŠTECHA, J. 2000, ÅSTRÖM, K. J. & WITTENMARK, B. 1995), but we will deal by this in the future; this is only a motivation example

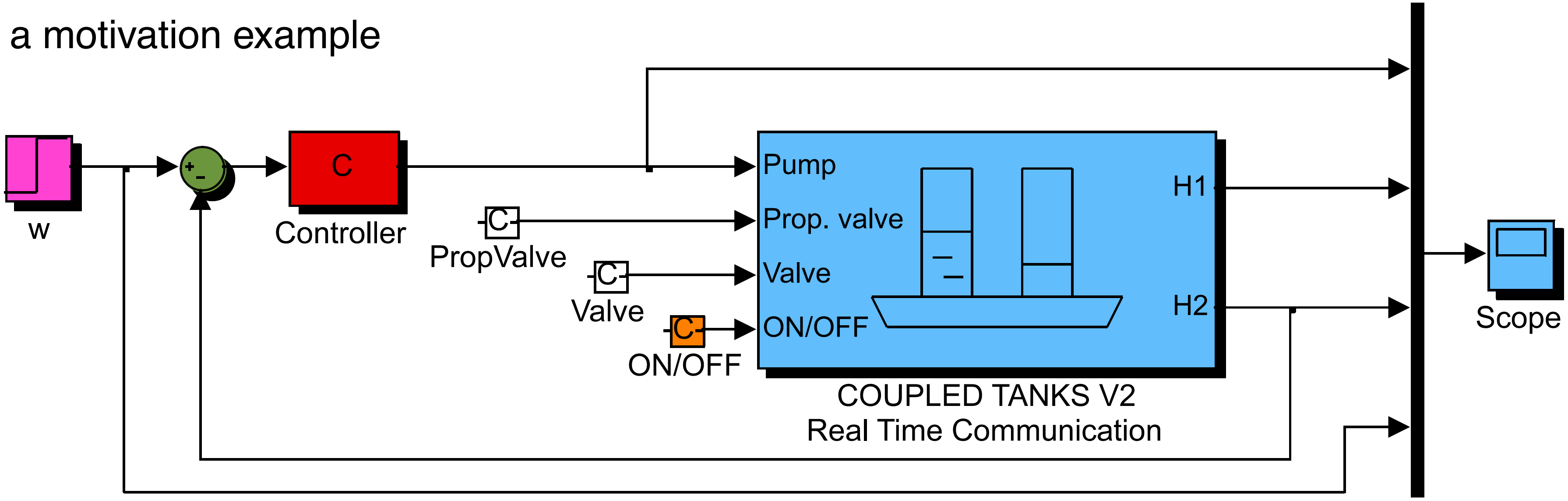


Figure 23: The closed loop with the real system

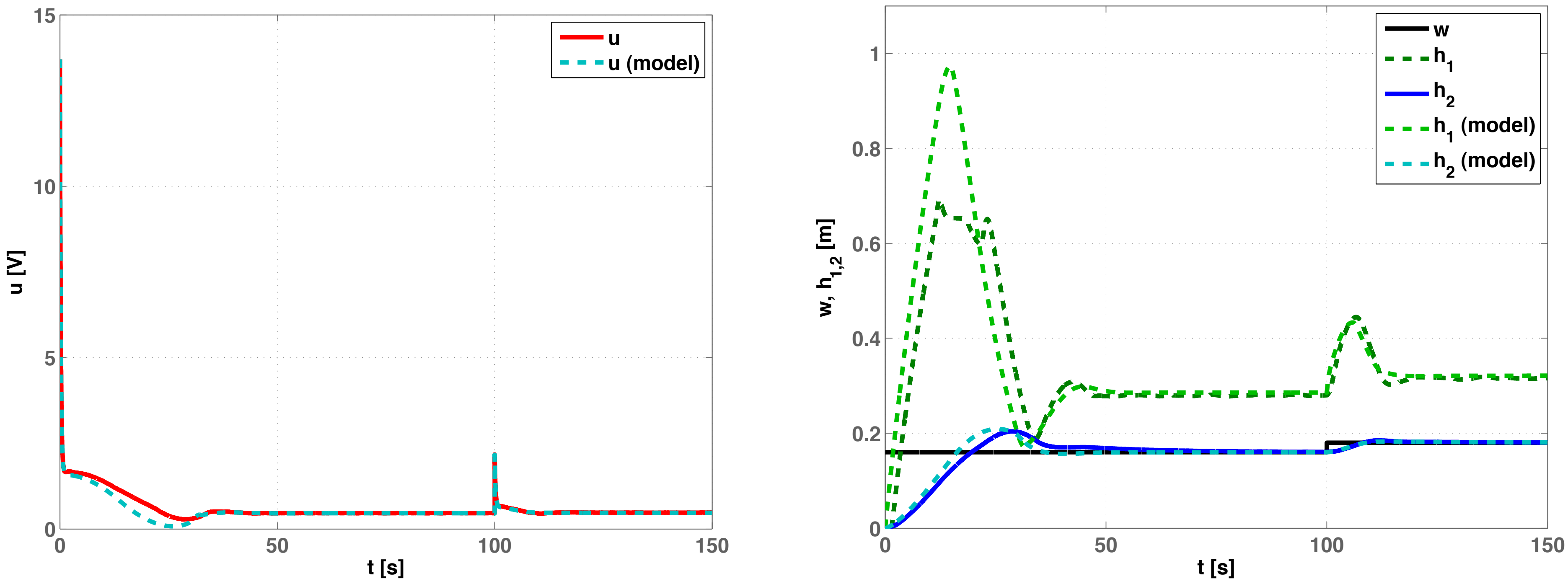


Figure 24: The system input $u(t)$ and the system output $y(t)$ time responses

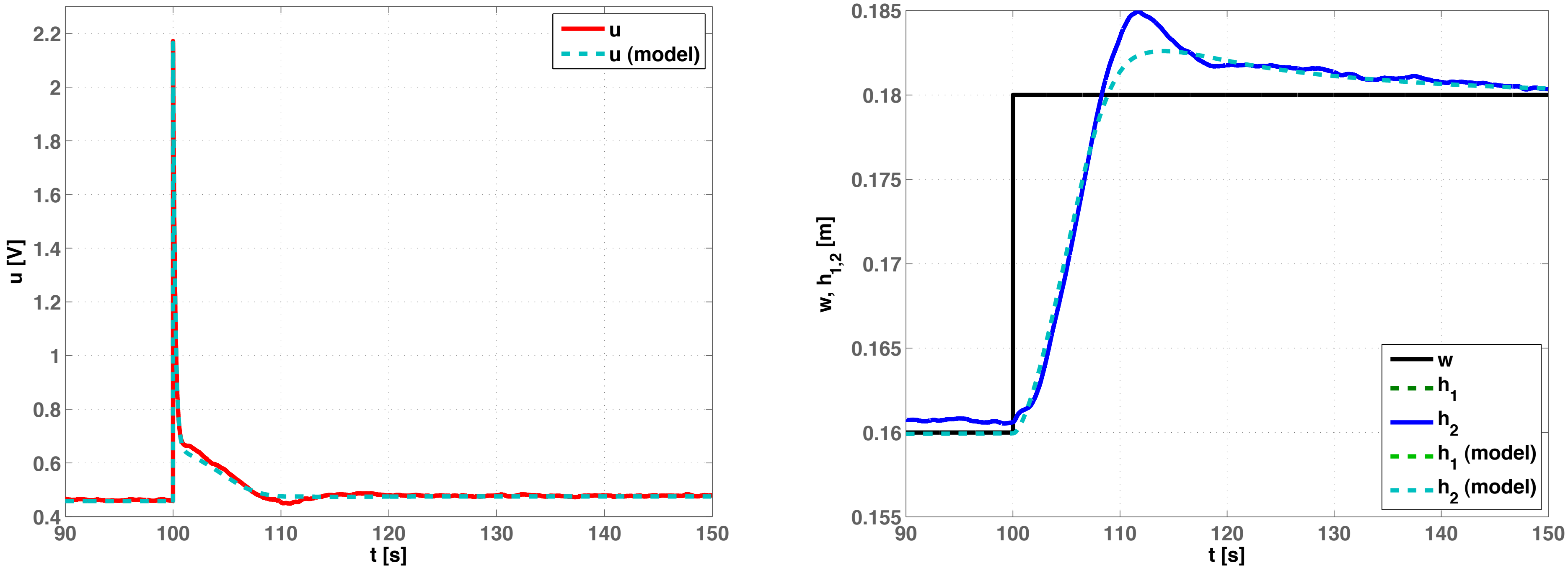


Figure 25: The system input $u(t)$ and the system output $y(t)$ time responses (zoom at the operating point)

3.1.10 Application of the LQ controller to the real system

- we designed other advance controller (ÅSTRÖM, K. J. & WITTENMARK, B. 1997, HAVLENA, V. & ŠTECHA, J. 2000, ÅSTRÖM, K. J. & WITTENMARK, B. 1995),
- of course, we have to simulate the system in the closed loop in the computer
- then, we can apply the LQ controller to the real system, see Figure 26, what is wrong in this scheme;
- the time responses of the system in the closed loop are in Figure 27 and Figure 28

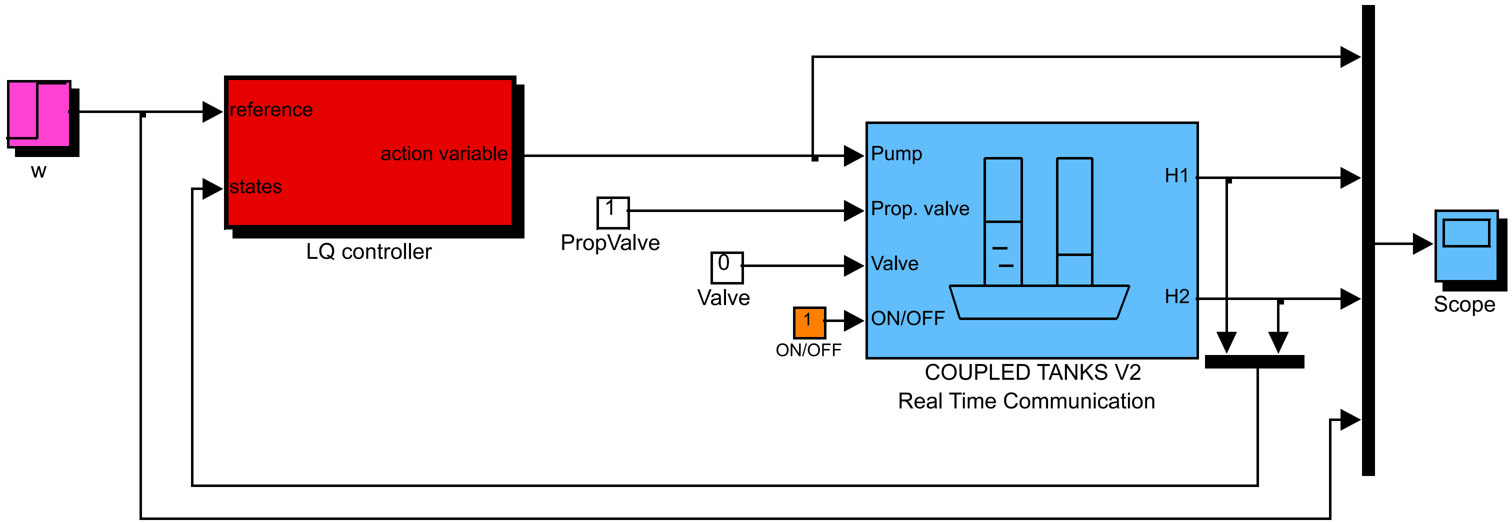


Figure 26: The closed loop with the real system and LQ controller

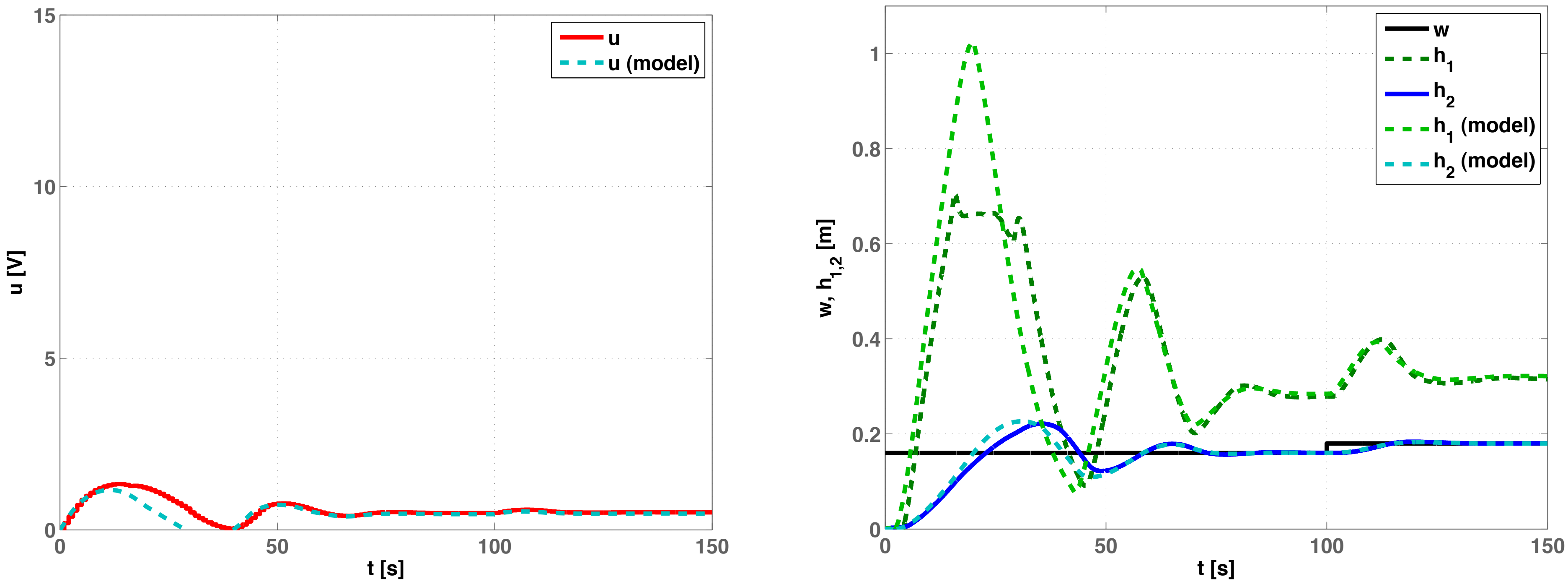


Figure 27: The system input $u(t)$ and the system output $y(t)$ time responses

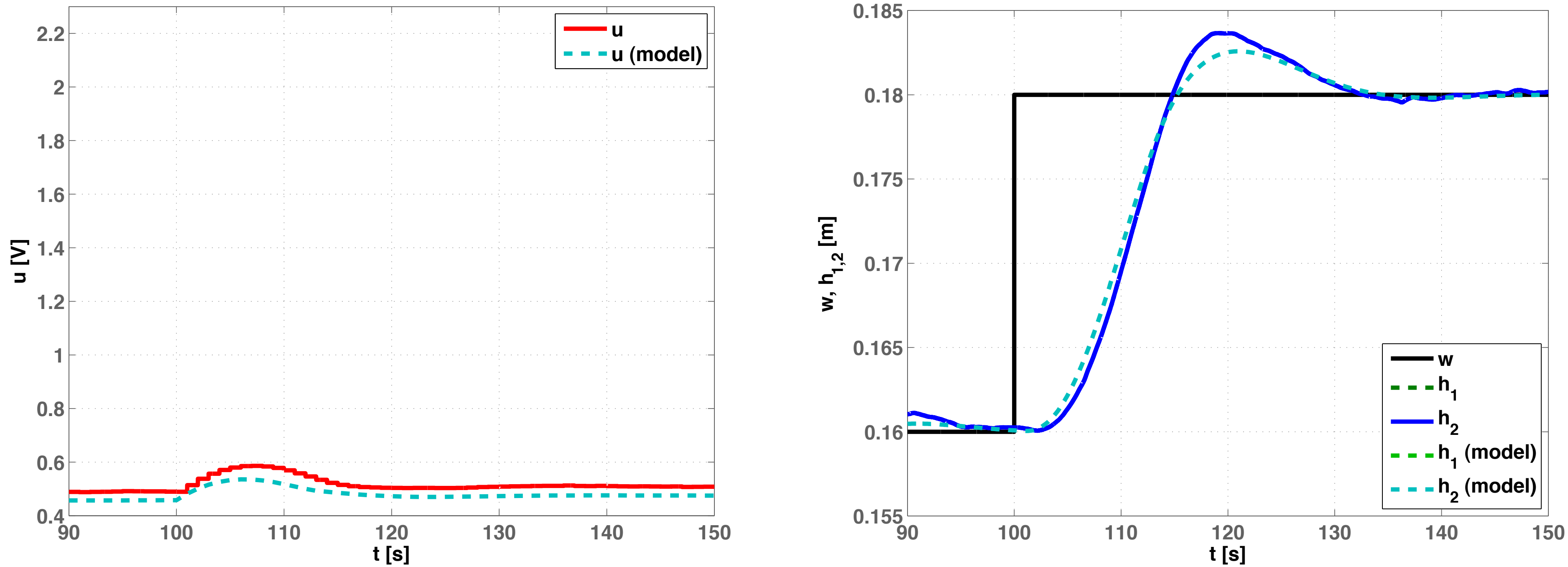


Figure 28: The system input $u(t)$ and the system output $y(t)$ time responses (zoom at the operating point)

4 Conclusion – and some Advices

- you can see it was a lot of work (a lot of steps) to reach what we want (to control the level in the right tank)
- a lot of students skip these steps during the control design and test the designed controller on the real system straightly,
- but they usually forget the operating points $u_{0\mathrm{lim}}$, $h_{10\mathrm{lim}}$, $h_{20\mathrm{lim}}$ at the linearization etc. and then their closed loop does not work correctly
- and then they think that the control theory is not right
- then they lose a lot of time to find the mistakes and finally they find out if they worked fair and went upon the mentioned procedure, they would save a lot of time even if they had not thought before
- we worked hard in the previous procedure and we were successful!
- $\Longrightarrow$ the fair work pays off in the end

Good luck with the control!