Discrete-time controllers structures and tuning

PID regulátory – spätnoväzbové štruktúry

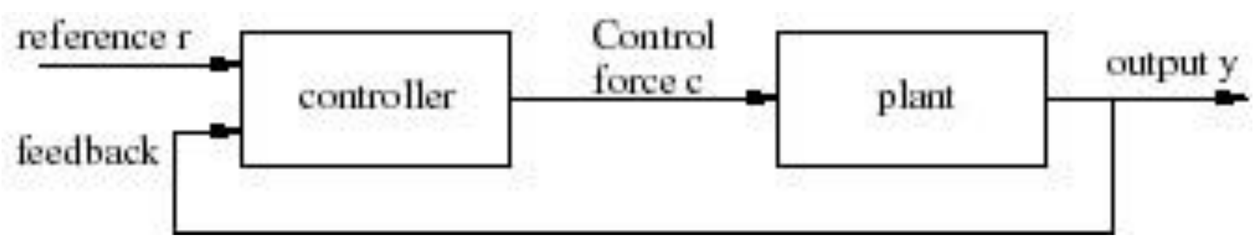
- Riadenie (CO) u(t) :

Proportional gain

$$u(t) = \left\{ K_p e(t) + K_I \int_0^t e(\tau)\,d\tau + K_d \frac{de}{dt} \right\}$$

Integral gain

Derivative gain



Proportional feedback gain K_P

- Proportional control : $u_p(t) = K_p e(t)$
- Feedback control c(t) is linearly proportional to the error : $e(t) = r(t) - y(t)$
- Steady state error will decrease
  $$e_{ss}(t) = \lim_{t \to \infty} \{e(t) = r(t) - y(t)\}$$
- Faster response
- Too much gain will make the system unstable

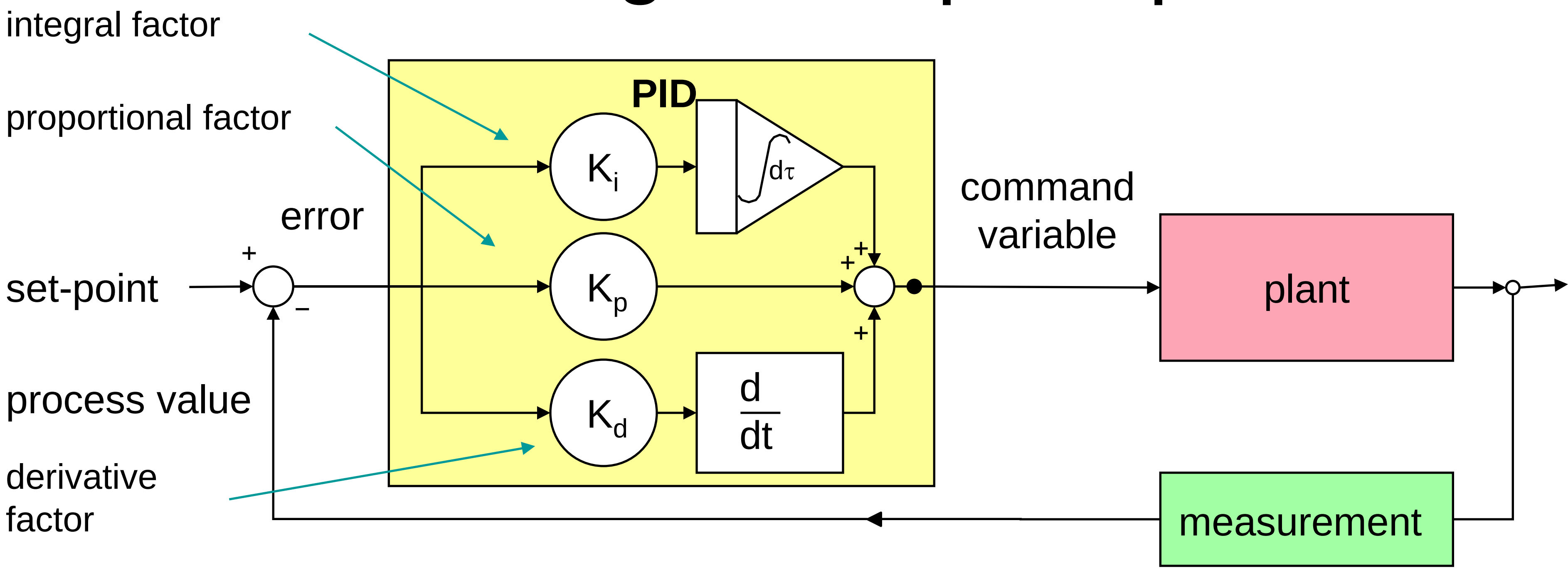
Integral feedback gain K_I

- Integral control: $$u_i(t) = K_I \int_0^t e(\tau)d\tau$$
- Penalty on the past error
- Zero steady state error
- Destabilizing influence
  - It gets oscillatory as K_I increases

Derivative feedback gain K_D

- Derivative control: $u_D(t) = K_D \frac{de}{dt}$
- Stabilize the system:
  – reduce oscillatory behavior
- Create a damping effect in the system dynamics
- It makes system slow down

Continuous regulator: principle of PID



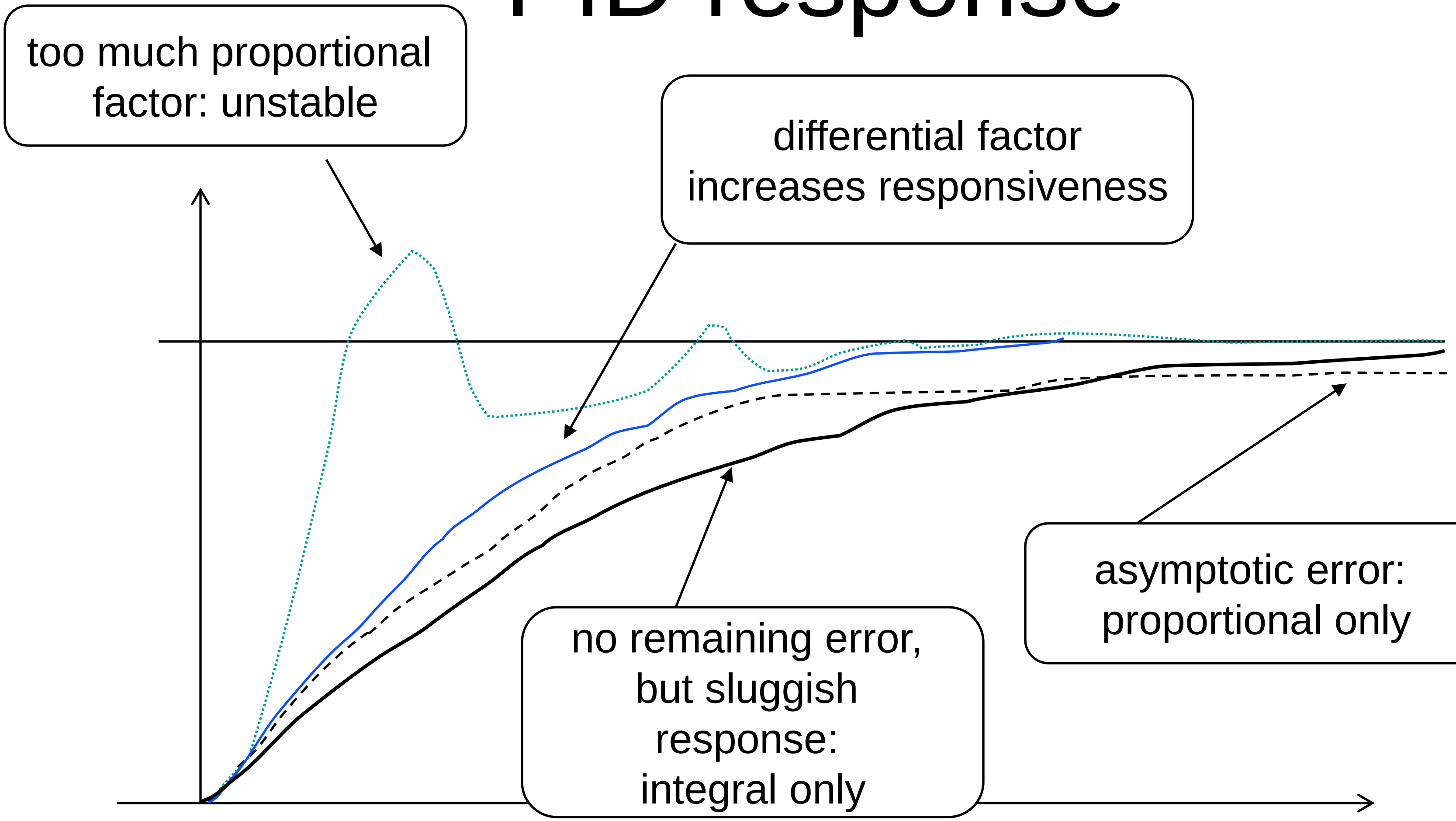
The proportional factor K_p generates an output proportional to the error, it requires a non-zero error to produce the command variable.

Increasing the amplification Kp decreases the error, but may lead to instability

The integral factor Ki produces a non-zero control variable even when the error is zero, but makes response slower.

The derivative factor K_d speeds up response by reacting to an error step with a control variable change proportional to the step.

PID response



Performance specifications of the closed loop system (step response)

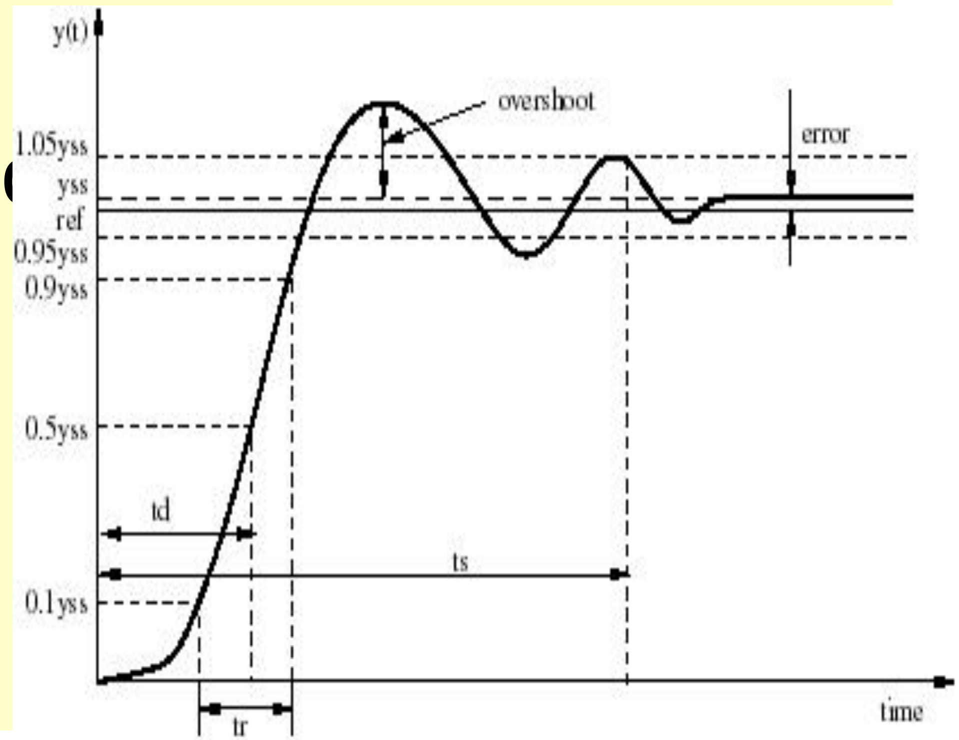
- Steady state error:

$$e_{ss} = \lim_{t \to \infty} e(t)$$

- Maximum oversh

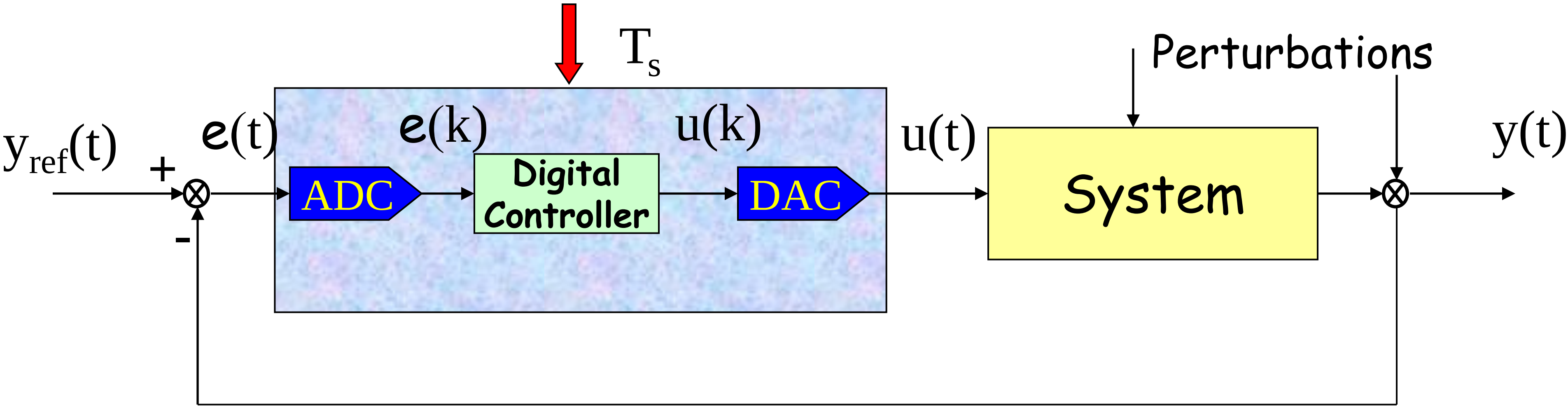
$$y_{max} - y_{ss}$$

- Delay time:
- Rise time:
- Settling time:



Digital control systems

Digital realisation of an "analogue type" controller

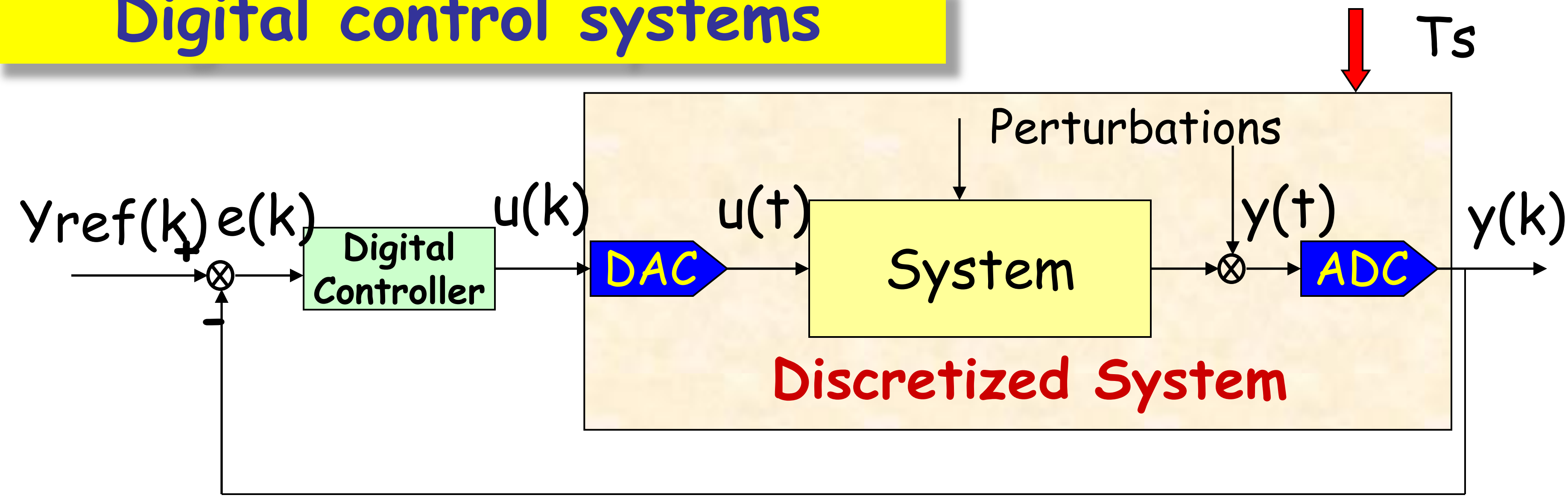


ADC- Digital controller - DAC should behave the same as an analogue controller (e.g. PID type), which implies **the use of a high sampling frequency** (the algorithm implemented is very simple)

Bad use of the potentialities of the digital controller

"Il ne suffit pas de mettre un TIGRE (microprocesseur ou DSP) dans son régulateur, il faut rajouter de l'intelligence"

Digital control systems



The sampling frequency is chosen in accordance with the bandwidth desired for the closed-loop system
Intelligent use of the "computer" : high sampling period and then implementation of complex algorithms requiring greater computation time.

Not only a copy of analogue control : BRAINWARE

Discrete-time system models and digital control algorithms

$y(k) = f[y(k-i), u(k-j)]$
or
$G(z^{-1}) = z^{-d} \; B(z^{-1})/A(z^{-1})$

Choice of sampling frequency

$$fs = 1/Ts = (6 \text{ to } 25) * f^{CL}_{B}$$

fs : sampling period

f^{CL}_{B} : bandwidth of the closed-loop system

If fs is fixed => limit for f^{CL}_{B} ($\cong fs/15$)

Úvod do prepočtov spojitých regulátorov na diskrétne formy

- Ideálne „textbook" PID regulátory
- Neideálne formy a opisy PID regulátorov
- Podmienky ekvivalentnosti spojitých a diskrétnych PID regulátorov vzhľadom na periódu vzorkovania
- Rekurentné formy – diferenčné rovnice diskrétnych PID regulátorov
- PID regulátory s ohraničením riadiaceho zásahu

Základné spojité formy PID regulátorov

$$u(t) = K\left[e(t) + \frac{1}{T_i}\int_0^t e(\tau)d\tau + T_d\frac{de(t)}{dt}\right]$$

$$G_R(s) = K\left[1 + \frac{1}{T_i s} + T_d(s)\right] = r_0 + \frac{r_{-1}}{s} + r_1 s$$

$$r_0 = K$$

$$r_{-1} = \frac{K}{T_i}$$

$$r_1 = KT_d$$

Neideálna forma opisu PID (realizovateľná)

$$G_R(s) = K\left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + T_f s}\right)$$

$$u(t) = K(e(t) + \frac{1}{T_i}\int e\,dt + \frac{T_d}{T_f}\frac{de}{dt}e^{-t/T_f})$$

ff =T_d*(1/T_f*Dirac(t)-1/T_f^2*exp(-t/T_f))

Neidealizovaný (reálny) PID regulátor obsahuje v derivačnej zložke oneskorovací člen (zabezpečujúci realizovateľnosť derivačnej zložky).

Základné diskrétne formy opisu PID regulátora

Prenosová funkcia diskrétneho regulátora v s a z-oblasti :

1.

$$G_R(s) = K\left(1 + \frac{1}{T_i s} + T_d s\right) = \frac{U(s)}{E(s)} \rightarrow G_R(z) = \frac{q_0 + q_1 z^{-1} + q_2 z^{-2}}{1 - z^{-1}} = \frac{U(z)}{E(z)}$$

$$u(k) = u(k-1) + q_0 e(k) + q_1 e(k-1) + q_2 e(k-2)$$

2.

$$G_R(z) = G_{TC} G_R(z) = \frac{z-1}{z} Z\left\{\frac{G_R(s)}{s}\right\}$$

$$G_R(s) = K\left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + T_f s}\right) = \frac{U(s)}{E(s)} \rightarrow G_R(z) = \frac{q_0 + q_1 z^{-1} + q_2 z^{-2}}{1 + p_1 z^{-1} + p_2 z^{-2}} = \frac{U(z)}{E(z)}$$

$$q_0 = K\left(1 + \frac{T_d}{T_f}\right)$$

$$q_1 = -K\left(1 - D_1 + 2\frac{T_d}{T_f} - \frac{T}{T_i}\right)$$

$$q_2 = K\left[\frac{T_d}{T_f} + \left(\frac{T_d}{T_i} - 1\right) D_1\right]$$

$$D_1 = -e^{-\frac{T}{T_f}}$$

$$p_1 = D_1 - 1$$

$$p_2 = -D_1$$

$$u(k) = -p_1 u(k-1) - p_2 u(k-2) + q_0 e(k) + q_1 e(k-1) + q_2 e(k-2)$$

Doplnok :

Ako určiť originál k derivačnej zložke

$$G_R(s) = K\left(1 + \frac{1}{T_i s} + \frac{T_d s}{1 + T_f s}\right) = \frac{U(s)}{E(s)}$$

$$L^{-1}\left\{\frac{T_d s}{1 + T_f s}\right\} = L^{-1}\left\{\frac{T_d}{T_f}\frac{s}{s + \frac{1}{T_f}}\right\} = L^{-1}\left\{\frac{T_d}{T_f}\frac{s + \frac{1}{T_f} - \frac{1}{T_f}}{s + \frac{1}{T_f}}\right\} = L^{-1}\left\{\frac{T_d}{T_f}\left(1 - \frac{\frac{1}{T_f}}{s + \frac{1}{T_f}}\right)\right\}$$

ff =T_d*(1/T_f*Dirac(t)-1/T_f^2*exp(-t/T_f))

Základné diskrétne formy PID regulátorov (DPID)

1. Ak nahradíme integrál v spojitej verzii sumou (obdlžníková náhrada) deriváciu diferenciou prvého rádu, potom v k-tom diskrétnom kroku riadiaci zásah je vyjadrený

$$u(k) = K\left[e(k) + \frac{T}{T_i}\sum_{i=1}^{k} e(i-1) + \frac{T_d}{T}\big(e(k) - e(k-1)\big)\right]$$

kde P - je koeficient zosilnenia odpovedajúci proporcionálnemu zosilneniu spojitého PID regulátora, T_i – resp. T_d sú koeficienty odpovedajúce integračnej resp. derivačnej časovej konštante spojitého regulátora

Rekurentný vzťah pre riadiaci zásah sa určí rozdielom u(k)-u(k-1)

k ⟶

$$u(k) = K\left[e(k) + \frac{T}{T_i}\sum_{i=1}^{k} e(i-1) + \frac{T_d}{T}\big(e(k) - e(k-1)\big)\right]$$

- **odčítaním**

k-1 ⟶

$$u(k-1) = K\left[e(k-1) + \frac{T}{T_i}\sum_{i=1}^{k-1} e(i-1) + \frac{T_d}{T}\big(e(k-1) - e(k-2)\big)\right]$$

$$\Delta u(k) = u(k) - u(k-1) = K\left[e(k) - e(k-1) + \frac{T}{T_i}e(k-1) + \frac{T_d}{T}\big(e(k) - 2e(k-1) + e(k-2)\big)\right]$$

$$\Delta u(k) = u(k) - u(k-1) = K\left[e(k) - e(k-1) + \frac{T}{T_i}e(k-1) + \frac{T_d}{T}\left(e(k) - 2e(k-1) + e(k-2)\right)\right] =$$

$$= K\left(1 + \frac{T_d}{T}\right)e(k) - K\left(1 + 2\frac{T_d}{T} - \frac{T}{T_i}\right)e(k-1) + K\frac{T_d}{T}e(k-2)$$

q_0 q_1 q_2

$$u(k) = u(k-1) + q_0 e(k) + q_1 e(k-1) + q_2 e(k-2)$$

$$q_0 = K\left(1 + \frac{T_d}{T}\right)$$

$$q_1 = -K\left(1 - \frac{T}{T_i} + 2\frac{T_d}{T}\right)$$

$$q_2 = K\frac{T_d}{T}$$

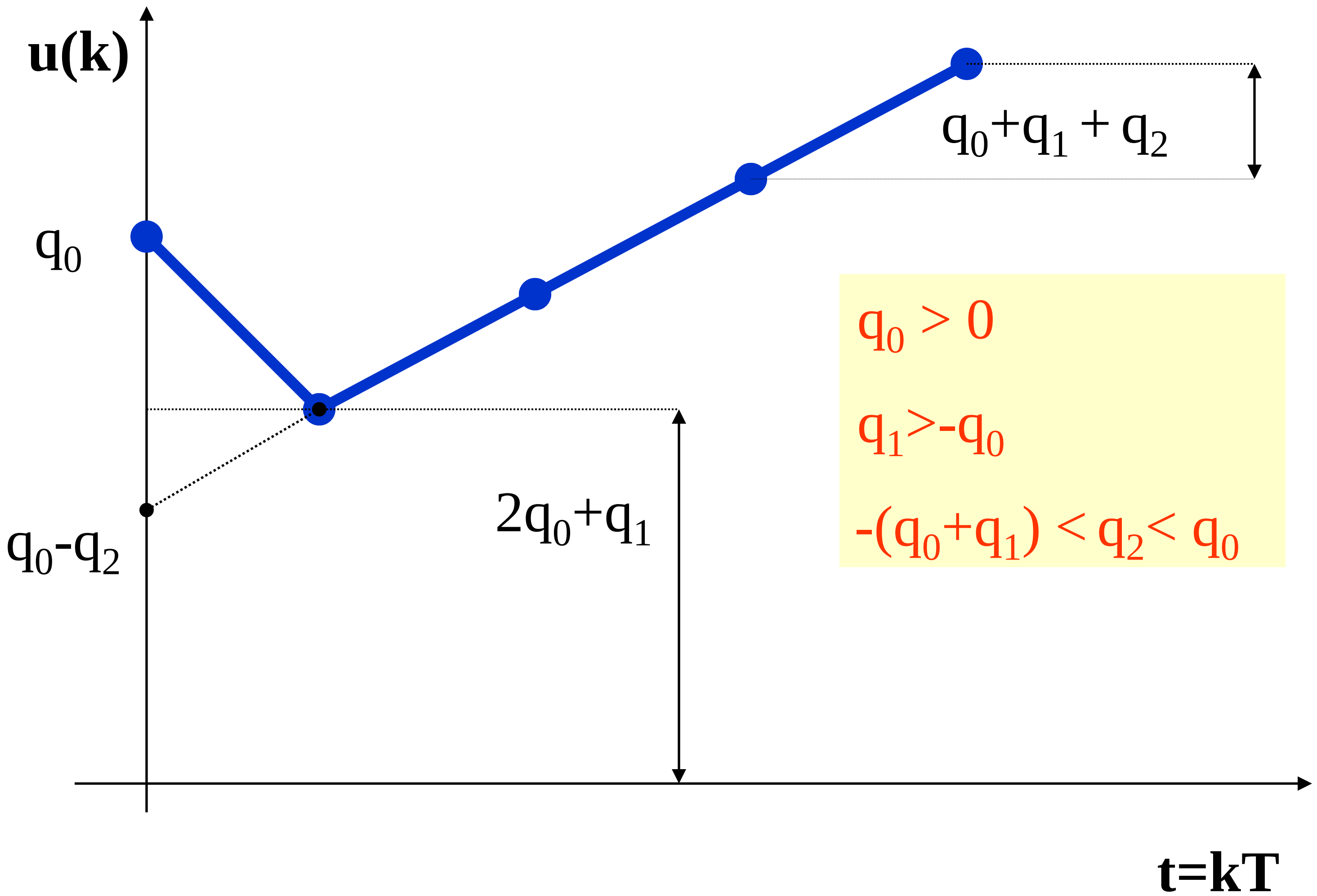
Podmienky ekvivalentnosti :

$q_0 > 0$

$q_1 > -q_0$

$-(q_0 + q_1) < q_2 < q_0$

PCH - PID REGULÁTORA-PODM.EKVIVALENTNOSTI



$q_0 > 0$

$q_1 > -q_0$

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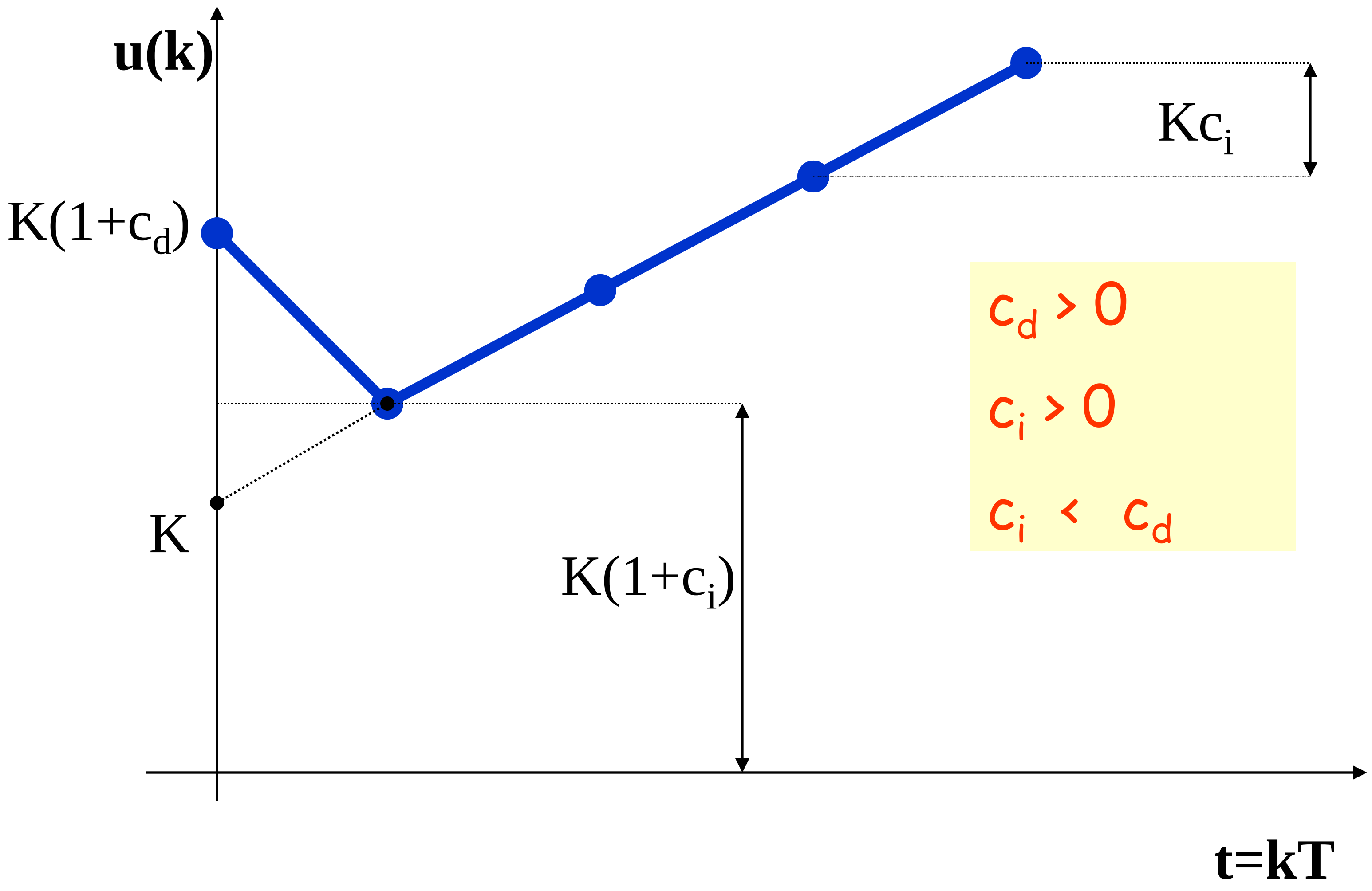
PCH - PID REGULÁTORA-PODM.EKVIVALENTNOSTI

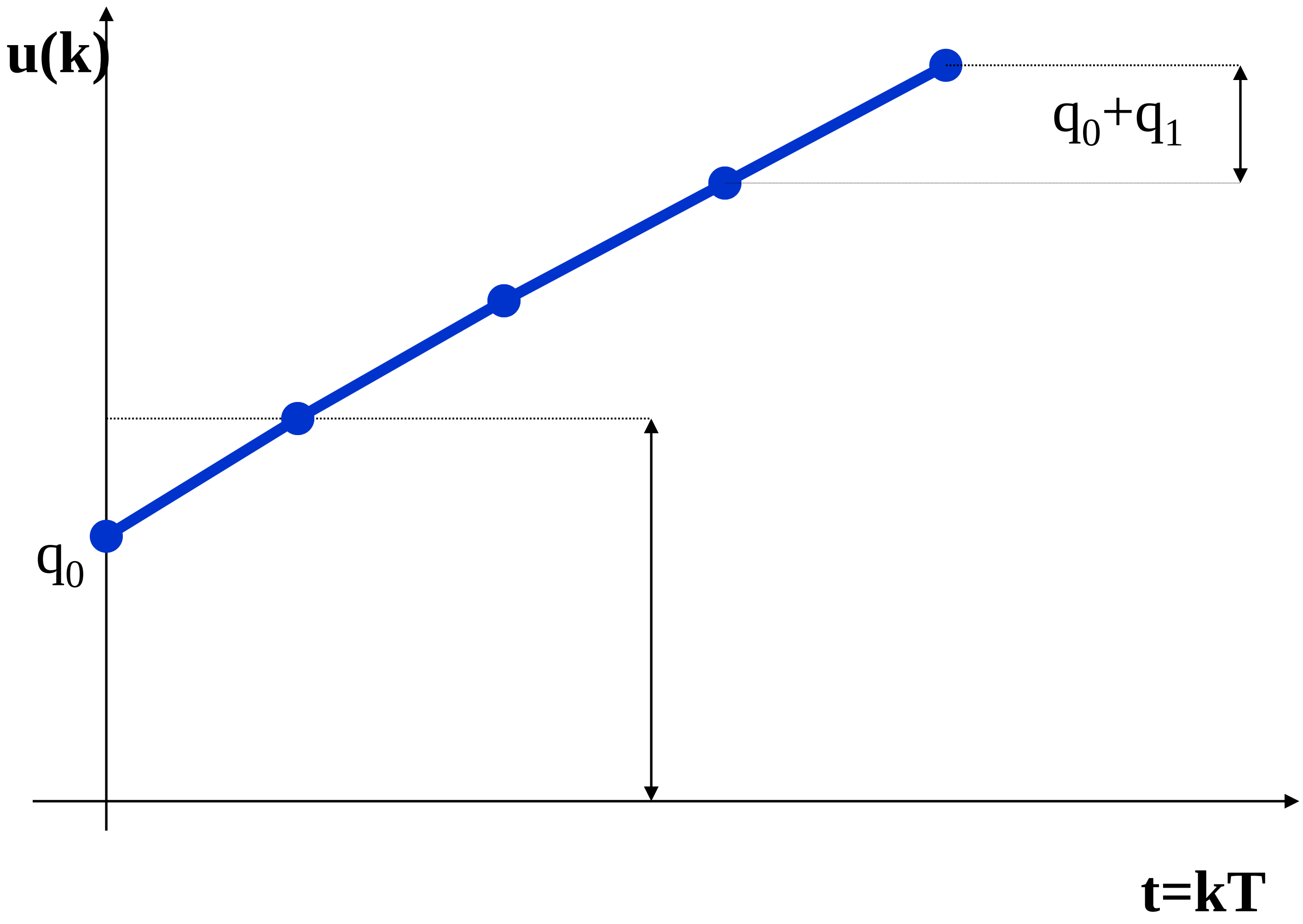


$c_d > 0$

$c_i > 0$

$c_i < c_d$

PCH - PI REGULÁTORA-PODM.EKVIVALENTNOSTI



Iné formy a vyjadrenia diskrétneho PID regulátora

$$\Delta u(k)=u(k)-u(k-1)=K\left[e(k)-e(k-1)+\frac{T}{T_i}e(k-1)+\frac{T_d}{T}\left(e(k)-2e(k-1)+e(k-2)\right)\right]=$$

$$=K\left(1+\frac{T_d}{T}\right)e(k)-K\left(1+2\frac{T_d}{T}-\frac{T}{T_i}\right)e(k-1)+K\frac{T_d}{T}e(k-2)$$

w(k)=w(k-1)=w(k-2)

$$u(k)=u(k-1)+K\left[\begin{array}{l} w(k)-y(k)-w(k-1)+y(k-1) \\ +\frac{T}{T_i}e(k-1)+ \\ \frac{T_d}{T}\left(w(k)-y(k)-2w(k-1)+2y(k-1)+w(k-2)-y(k-2)\right) \end{array}\right]=$$

$$=u(k-1)+K[-y(k)+y(k-1)]+K\frac{T}{T_i}e(k-1)+K\frac{T_d}{T}[-y(k)+2y(k-1)-y(k-2)]$$

Takahashiho vzt'ah (feedforward forma diskrétneho PID-u):

$$u(k)=u(k-1)+K[-y(k)+y(k-1)]+K\frac{T}{T_i}e(k-1)+K\frac{T_d}{T}[-y(k)+2y(k-1)-y(k-2)]$$

2. Ak nahradíme integrál v spojitej verzii sumou (lichobežníková náhrada) deriváciu diferenciou prvého rádu, potom v k-tom a k-1 diskrétnom kroku riadiaci zásah je vyjadrený

$$u(k) = K\left[e(k) + \frac{T}{T_i}[(\frac{e(0)+e(k)}{2}) + \sum_{i=1}^{k-1} e(i)] + \frac{T_d}{T}\left(e(k) - e(k-1)\right)\right]$$

-

$$u(k-1) = K\left[e(k-1) + \frac{T}{T_i}[(\frac{e(0)+e(k-1)}{2}) + \sum_{i=1}^{k-2} e(i)] + \frac{T_d}{T}\left(e(k-1) - e(k-2)\right)\right]$$

$$u(k) = u(k-1) + K\left[e(k)(1 + \frac{T}{2T_i} + \frac{T_d}{T}) + e(k-1)(1 + 2\frac{T_d}{T} - \frac{T}{2T_i}) + e(k-2)\frac{T_d}{T}\right]$$

q_0 q_1 q_2

$$u(k) = u(k-1) + q_0 e(k) + q_1 e(k-1) + q_2 e(k-2)$$

$$q_0 = K\left(1 + \frac{T}{2T_i} + \frac{T_d}{T}\right) \qquad q_1 = -K\left(1 + 2\frac{T_d}{T} - \frac{T}{2T_i}\right) \qquad q_2 = K\frac{T_d}{T}$$

Prenosová funkcia diskrétneho PID regulátora

$$\Delta u(k) = q_0 e(k) + q_1 e(k-1) + q_2 e(k-2)$$

$$Z\{u(k)\} - Z\{u(k-1)\} = Z\{q_0 e(k)\} + Z\{q_1 e(k-1)\} + Z\{q_2 e(k-2)\}$$

$$U(z) - z^{-1} U(z) = q_0 E(z) + q_1 z^{-1} E(z) + q_2 z^{-2} E(z)$$

$$U(z)(1 - z^{-1}) = (q_0 + q_1 z^{-1} + q_2 z^{-2}) E(z)$$

$$G_R(z) = \frac{q_0 + q_1 z^{-1} + q_2 z^{-2}}{1 - z^{-1}} = \frac{Q(z)}{P(z)} = \frac{U(z)}{E(z)}$$

Podmienky ekvivalentnosti PID a PSD regulátora

$$u(k) = u(k-1) + q_0 e(k) + q_1 e(k-1) + q_2 e(k-2)$$

$$e(k) = 1(k) = \begin{cases} 1 & pre \quad k \geq 0 \\ 0 & pre \quad k < 0 \end{cases}$$

$$k = 0 \quad u(0) = q_0$$

$$k = 1 \quad u(1) = u(0) + q_0 + q_1 = 2q_0 + q_1$$

$$k = 2 \quad u(2) = u(1) + q_0 + q_1 + q_2 = 3q_0 + 2q_1 + q_2$$

....

$$u(k) = u(k-1) + q_0 + q_1 + q_2 = (k+1)q_0 + kq_1 + (k-1)q_2$$

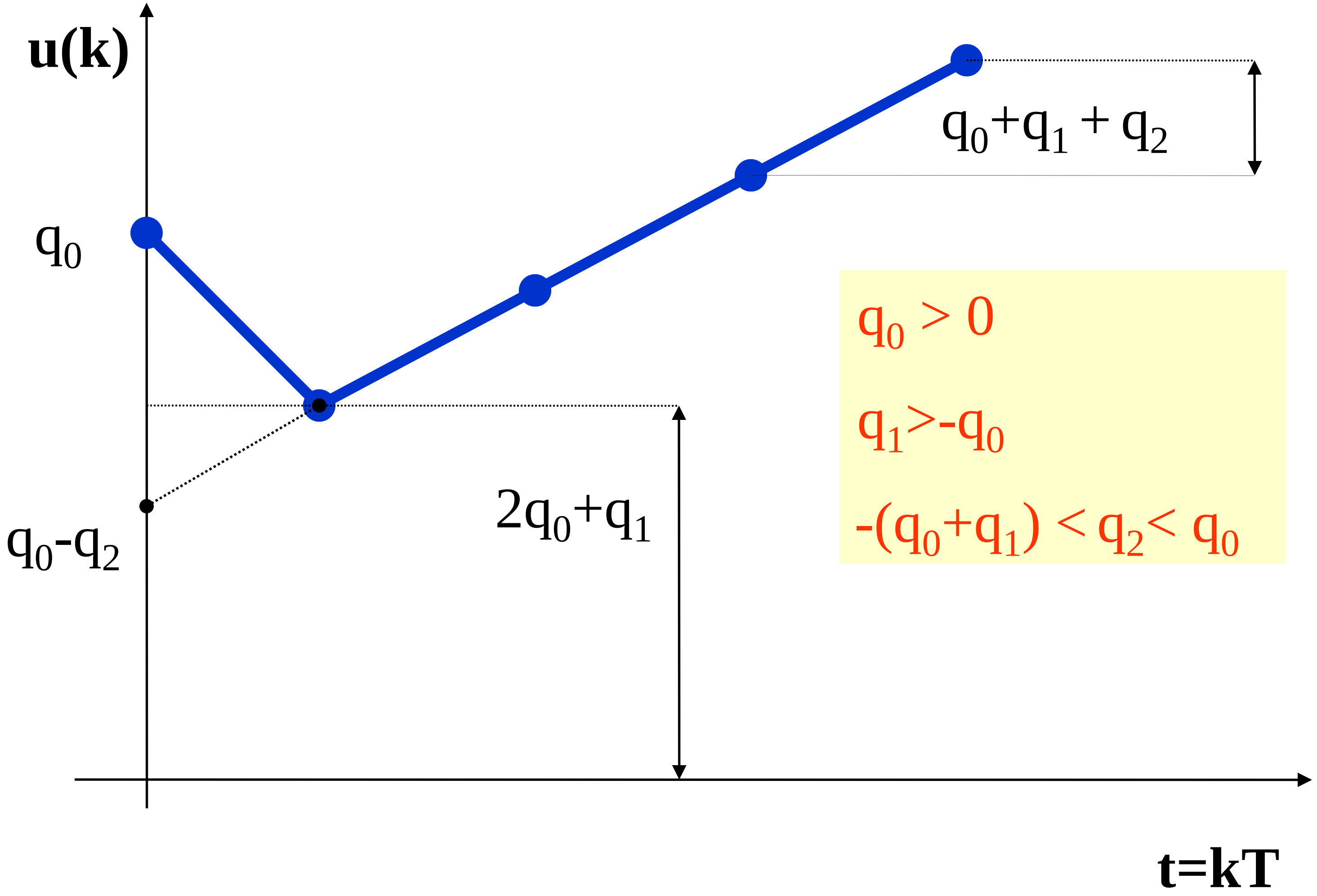
$u(1) < u(0) \qquad 2q_0 + q_1 < q_0 \qquad q_0 + q_1 < 0 \qquad q_1 < -q_0$ $\qquad q_0 > 0$

$u(k) > u(k-1) \quad pre \quad k \geq 2$

$u(k) > u(k-1) \quad pre \ k \geq 2 \qquad q_0 + q_1 + q_2 > 0 \ \ alebo \ \ q_2 > -(q_0 + q_1)$

Podmienky „**ekvivalentnosti**“:

$$q_0 > 0 \qquad q_1 < -q_0 \qquad -(q_0 + q_1) < q_2 < q_0$$



u(k)
q_0
q_0-q_2
$2q_0+q_1$
$q_0+q_1 + q_2$
$q_0 > 0$
$q_1>-q_0$
$-(q_0+q_1) < q_2< q_0$
t=kT

Iná ekvivalentná forma vyjadrenia diskrétneho PID regulátora

$$K = q_0 - q_2$$

$$c_d = \frac{q_2}{K} \quad \rightarrow \quad c_d = \frac{T_d}{T} \quad \rightarrow \quad K_d = K\frac{T_d}{T} = Kc_d$$

$$c_i = \frac{(q_0 + q_1 + q_2)}{K} \quad \rightarrow \quad c_i = \frac{T}{T_i} \quad \rightarrow \quad K_i = K\frac{T_i}{T} = Kc_i$$

$$G_R(z) = \frac{K\left[(1 + c_d) + (c_i - 2c_d - 1)z^{-1} + c_d z^{-2}\right]}{1 - z^{-1}} =$$

$$= K\left[1 + c_i \frac{z^{-1}}{1 - z^{-1}} + c_d(1 - z^{-1})\right] = \frac{U(z)}{E(Z)} = \frac{q_0 + q_1 z^{-1} + q_2 z^{-2}}{1 - z^{-1}} \qquad \text{(rkoz0)}$$

Podmienky ekvivalentnosti PSD regulátora s PID regulátorom

$$c_d > 0 \qquad c_i > 0 \qquad c_i < c_d$$

$$G_R(z) = \frac{K\left[(1 + c_d + c_i) + (-2c_d - 1)z^{-1} + c_d z^{-2}\right]}{1 - z^{-1}} =$$

$$= K\left[1 + c_i \frac{z^{-1}}{1 - z^{-1}} + c_d(1 - z^{-1})\right] = \frac{U(z)}{E(Z)} = \frac{q_0 + q_1 z^{-1} + q_2 z^{-2}}{1 - z^{-1}}$$

Upravený tvar DPID

Veľmi často sa odchýlka nahrádza e(k)← e(k-1), čím sa dosahuje okamžité pôsobenie riadiaceho zásahu na proces. Táto zmena sa prejaví aj v prenosovej funkcii regulátora na integračnej zložke, ktorá neobsahuje v čitateli člen z^{-1}.

$$G_R(z) = \frac{K\left[(1+c_d+c_i)+(-2c_d-1)z^{-1}+c_d z^{-2}\right]}{1-z^{-1}} =$$

$$= K\left[1+c_i\frac{1}{1-z^{-1}}+c_d(1-z^{-1})\right] = \frac{U(z)}{E(Z)} = \frac{q_0+q_1z^{-1}+q_2z^{-2}}{1-z^{-1}}$$

$$K = q_0 - q_2$$

$$c_d = \frac{q_2}{K}$$

$$c_i = \frac{(q_0+q_1+q_2)}{K}$$

Paralelná forma diskrétneho PID regulátora:

Riadiaci zásah podľa (rkoz0) je potom tvorený súčtom jednotlivých zložiek

$$u(k) = u_p(k) + u_i(k) + u_d(k)$$

Proporcionálna zložka: $u_p(k) = Ke(k) = (q_0 - q_2)e(k)$

Integračná zložka: $u_i(k) = u_i(k-1) + Kc_i e(k-1) = u_i(k-1) + (q_0+q_1+q_2)\, e(k-1)$

Derivačná zložka: $u_d(k) = Kc_d e(k) - K c_d e(k-1) = q_2[e(k) - e(k-1)]$

Vynechávaním jednotlivých koeficientov q_i, pre i=0,1,2 dostaneme rôzne štruktúry diskrétnych regulátorov.

Ak vo vzťahu (rkoz0) položíme q2=0, dostaneme prenosovú funkciu diskrétneho regulátora v tvare:

$$G_R(z) = \frac{q_0 + q_1 z^{-1}}{1 - z^{-1}} = \frac{U(z)}{E(z)} \qquad \text{(rkoz1)}$$

Diskrétny regulátor opísaný vzťahom (rkoz1) voláme diskrétny regulátor prvého rádu (PS-regulátor).

Riadiaci zásah diskrétneho PI regulátora je vyjadrený diferenčnou rovnicou

$$E(z)(q_0 + q_1 z^{-1}) = (1 - z^{-1})U(z)$$

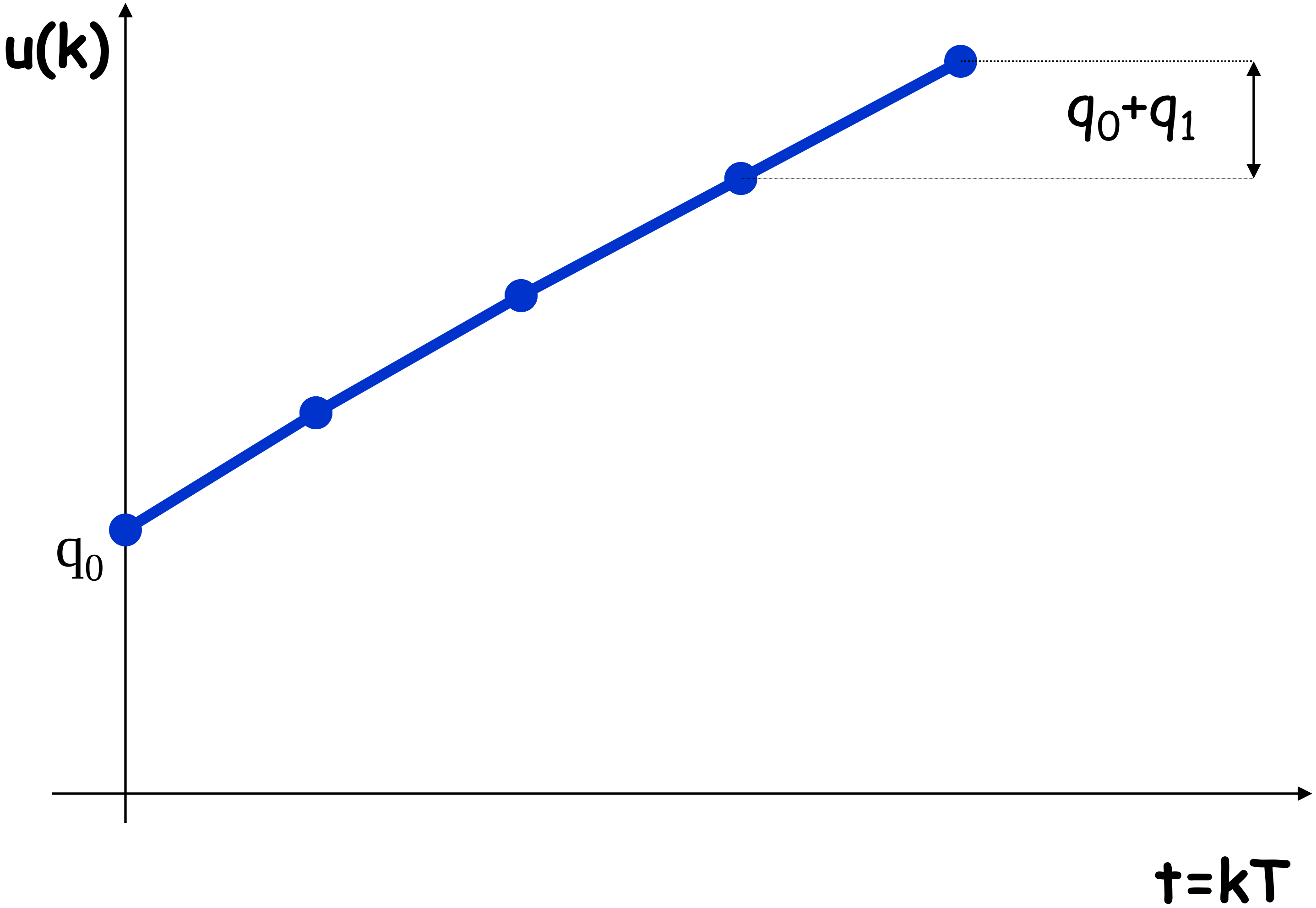
$$u(k) = u(k-1) + q_0 e(k) + q_1 e(k-1)$$

Podmienky ekvivalentnosti sú odvodené podobne ako u PID regulátora

$$u(1) > u(0) \quad a \quad q_0 > 0$$

$$q_0 + q_1 > 0 \quad alebo \quad q_1 > -q_0$$

PCH - PI REGULÁTORA-PODM.EKVIVALENTNOSTI



Iné vyjadrenie PS regulátora je možné pomocou koeficientov K, c_i a c_d. Pre q_2=0 je zosilnenie K a koeficienty c_d a c_i vyjadrené

$$K = q_0$$
$$c_d = 0$$
$$c_i = K(q_0 + q_1)$$

$$\mathrm{K} > 0 \qquad (\mathrm{q}_0 > 0)$$

$$\mathrm{c_i} > 0$$

Prenosová funkcia diskrétneho PI regulátora (DPI) použitím koeficientov K, c_i :

$$G_R(z) = \frac{K\left[1 + (c_i - 1)z^{-1}\right]}{1 - z^{-1}} = \frac{U(Z)}{E(z)}$$

Riadiaci zásah

$$u(k) = u(k-1) + Ke(k) + K(c_i - 1)e(k-1)$$

Diskrétny I regulátor získame ak položíme $q_0=0$, $q_2=0$. Prenosová funkcia diskrétneho I regulátora je v tvare:

$$G_R(z) = \frac{q_1 z^{-1}}{1 - z^{-1}} = \frac{U(z)}{E(z)}$$

Riadiaci zásah určíme z prenosovej funkcie $u(k) = u(k-1) + q_1 e(k-1)$

Ak položíme $c_i=0$, dostaneme diskrétny PD regulátor s prenosovou funkciou:

$$G_R(z) = q_0 - q_2 z^{-1} = \frac{U(z)}{E(z)} = K\left[1 + c_d(1 - z^{-1})\right]$$

$$K = (q_0 - q_2)$$

$$c_d = \frac{q_2}{K}$$

Prenosová funkcia **diskrétneho P** regulátora $G_R(z) = q_0$

Riadiaci zásah P regulátora: $u(k) = q_0 e(k)$? $u(k) = u(k-1) + K[e(k) - e(k-1)]$ ✓

$$\Delta u(k) = u(k) - u(k-1) = P\left[e(k) - e(k-1) + \frac{T}{T_i} e(k-1) + \frac{T_d}{T}\big(e(k-1) - 2e(k-1) + e(k-2)\big)\right]$$

Modifikácia PID regulátorov úpravou derivačného člena

- jednoduchá náhrada derivácie diferenciou prvého rádu vnáša nepresnosti do rekurzívnzych a nerekurzívnzych foriem PSD regulátorov a môže spôsobiť, že riadiaci zásah nadobúda veľké a prudké zmeny.
- Aby sa tomu predišlo, využíva sa náhrada derivácie priemernou hodnotou napr. zo štyroch hodnôt odchýlky:

$$e_s(k) = \frac{1}{4}\sum_{i=k-3}^{k} e(i) = \frac{e(k-3)+e(k-2)+e(k-1)+e(k)}{4}$$

- Ak použijeme nerekurzívnu formu PID regulátora, potom deriváciu nahradíme vzťahom:

$$T_d \frac{de}{dt} = \frac{T_d}{T}\Delta e_s(k) =$$

$$= \frac{T_d}{4}\left[\frac{e(k)-e_s(k)}{1.5T} + \frac{e(k-1)-e_s(k)}{0.5T} + \frac{e_s(k)-e(k-3)}{0.5T} + \frac{e_s(k)-e(k-3)}{1.5T}\right] =$$

$$= \frac{T_d}{6T}[e(k)+3e(k-1)-3e(k-2)-e(k-3)]$$

pre rekurentnú formu:

$$T_d \frac{de}{dt} = \frac{T_d}{6T}[e(k)+2e(k-1)-6e(k-2)+2e(k-3)+e(k-4)]$$

PID formy „neidealizovaného“ diskrétneho regulátora

- Ak spojitý PID regulátor obsahuje v derivačnej zložke oneskorovací člen, môžeme jeho diskrétny opis určiť niekoľkými spôsobmi. Prakticky sa využívajú dva spôsoby prepočtu:
- Prvý spôsob prepočtu je realizovaný na základe určenia z-obrazu zo spojitého opisu, t.j.

$$G_R(z) = G_{TC}G_R(z) = \frac{z-1}{z} Z\left\{\frac{G_R(s)}{s}\right\} \qquad G_R(s) = K\left[1 + \frac{1}{T_i s} + \frac{T_d s}{1 + T_f s}\right]$$

$$G_R(z) = K\frac{z-1}{z} K\, Z\left[L^{-1}\left\{\frac{1}{s} + \frac{1}{T_i s^2} + \frac{T_d}{1 + T_{fi} s}\right\}\right] = K\left[1 + \frac{T}{T_i}\cdot\frac{1}{z-1} + \frac{T_d}{T_f}\frac{z-1}{z+D_1}\right] =$$

$$= K\left[1 + \frac{T}{T_i}\cdot\frac{z^{-1}}{1-z^{-1}} + \frac{T_d}{T_f}\cdot\frac{1-z^{-1}}{1+D_1 z^{-1}}\right] =$$

$$= \frac{K\left[\left(1 + \frac{T_d}{T_f}\right)z^0 - \left(1 - D_1 + 2\frac{T_d}{T_f} - \frac{T}{T_i}\right)z^{-1} + \left(\frac{T_d}{T_f} + \left(\frac{T}{T_i} - 1\right)D_1\right)z^{-2}\right]}{\left(1 - z^{-1}\right)\left(1 + D_1 z^{-1}\right)} = \frac{q_0 + q_1 z^{-1} + q_2 z^{-2}}{1 + p_1 z^{-1} + p_2 z^{-2}} = \frac{U(z)}{E(z)}$$

$$G_R(z)=\frac{K\left[\left(1+\frac{T_d}{T_f}\right)z^0-\left(1-D_1+2\frac{T_d}{T_f}-\frac{T}{T_i}\right)z^{-1}+\left(\frac{T_d}{T_f}+\left(\frac{T}{T_i}-1\right)D_1\right)z^{-2}\right]}{\left(1-z^{-1}\right)\left(1+D_1z^{-1}\right)}=\frac{q_0+q_1z^{-1}+q_2z^{-2}}{1+p_1z^{-1}+p_2z^{-2}}=\frac{U(z)}{E(z)}$$

$$q_0=K\left(1+\frac{T_d}{T_f}\right)$$

$$q_1=-K\left(1-D_1+2\frac{T_d}{T_f}-\frac{T}{T_i}\right)$$

$$q_2=K\left[\frac{T_d}{T_f}+\left(\frac{T_d}{T_i}-1\right)D_1\right]$$

$$D_1=-e^{-\frac{T}{T_f}}$$

$$p_1=D_1-1$$

$$p_2=-D_1$$

$$u(k)=-p_1u(k-1)-p_2u(k-2)+q_0e(k)+q_1e(k-1)+q_2e(k-2)$$

Druhý spôsob výpočtu parametrov PID neideálneho diskrétneho PID regulátora môžeme určiť aproximatívnym spôsobom podľa Tustinového vzťahu.

Dosadením za $s=\frac{2(z-1)}{T(z+1)}$ → $G_R(s)=K\left[1+\frac{1}{T_is}+\frac{T_ds}{1+T_fs}\right]$

↓

$$G_R(z)=\frac{q_o+q_1z^{-1}+q_2z^{-2}}{1+p_1z^{-1}+p_2z^{-2}}=\frac{U(z)}{E(z)}$$

$$G_R(z) = \frac{q_o + q_1 z^{-1} + q_2 z^{-2}}{1 + p_1 z^{-1} + p_2 z^{-2}} = \frac{U(z)}{E(z)}$$

$$q_0 = \frac{K[1 + 2(c_{p1} + c_{d1} + 0.5 \cdot c_{p1}(1 + 2c_{p1}))]}{1 + 2c_{p1}}$$

$$q_1 = \frac{K[c_{p1} - 4(c_{p1} + c_{d1})]}{1 + 2c_{p1}}$$

$$q_2 = \frac{K[c_{p1}(2 - c_{i1}) + 2c_{d1} + 0.5 \cdot c_{i1} - 1]}{1 + 2c_{p1}}$$

$$c_{p1} = \frac{T_f}{T}$$

$$c_{i1} = \frac{T}{T_i}$$

$$c_{d1} = \frac{T_d}{T}$$

$$p_1 = -4c_{p1}(1 + 2c_{p1})$$

$$p_2 = \frac{2c_{p1} - 1}{1 + 2c_{p1}}$$

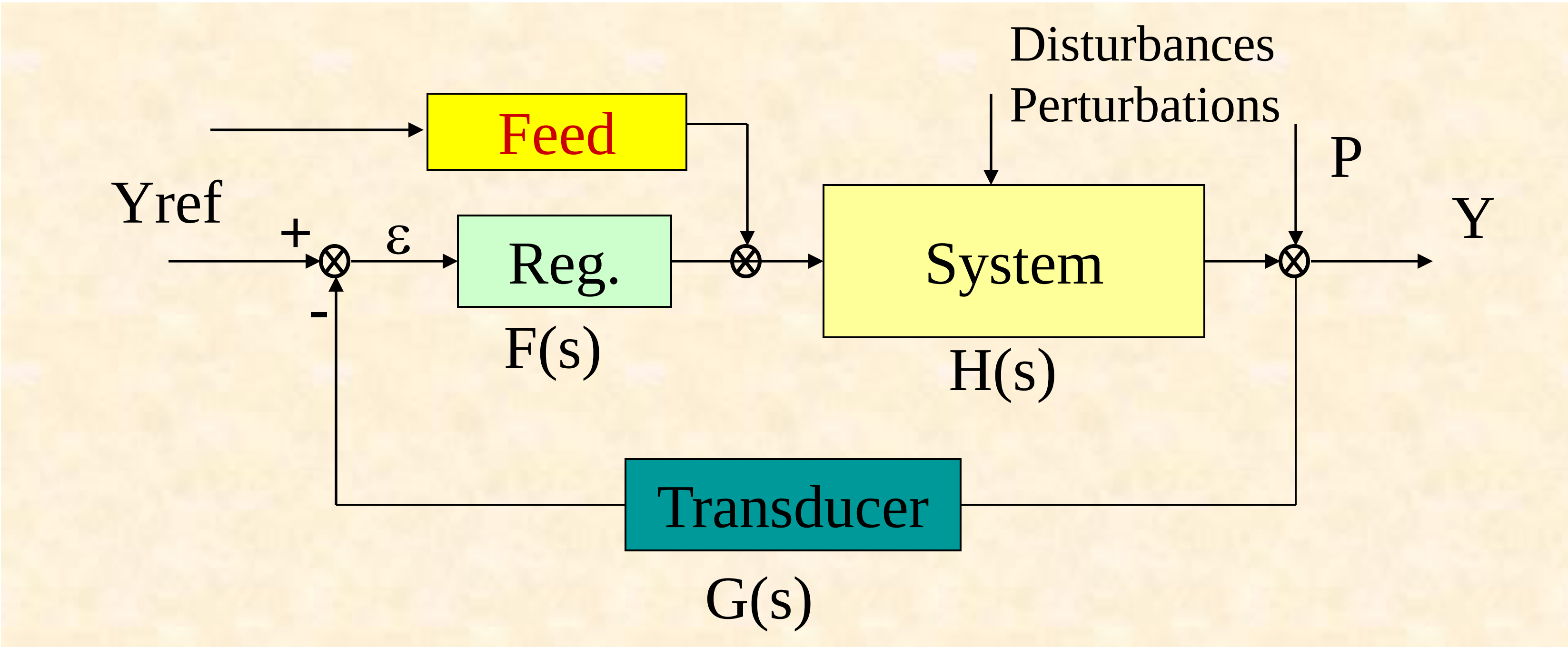
Riadiaci zásah PID(NI) regulátora

$$u(k) = -p_1 u(k-1) - p_2 u(k-2) + q_0 e(k) + q_1 e(k-1) + q_2 e(k-2)$$

Typ	Koeficienty PSD regulátora		$G_R(z)=\dfrac{q_0+q_1z^{-1}+q_2z^{-2}}{p_0-p_1z^{-1}-p_2z^{-2}}$
	q_0	q_1	q_2
	p_0	p_1	p_2
PSD (1)	$K\left(1+\dfrac{T_d}{T}\right)$	$-K\left(1+2\dfrac{T_d}{T}-\dfrac{T}{T_i}\right)$	$K\dfrac{T_d}{T}$
	1	1	1
PSD (2)	$K\left(1+\dfrac{T}{2T_i}+\dfrac{T_d}{T}\right)$	$-K\left(1+\dfrac{2T_d}{T}-\dfrac{T}{2T_i}\right)$	$K\dfrac{T_d}{T}$
	1	1	0
PSD (3)	$K\left(1+\dfrac{T_d}{T_0}\right)$	$-K\left(1-D_1+\dfrac{2T_d}{T_0}-\dfrac{T}{T_i}\right)$ $D_1=-\exp(-T/T_i)$	$K\left(\dfrac{T_d}{T_0}+\dfrac{T}{T_i}-1\right)D_1$
	1	$D_1=1$	$-D_1$
PSD (4)	$\dfrac{K\left[1+2(c_{p1}+c_{d1})+c_{i1}/2(1+2c_{p1})\right]}{1+2c_{p1}}$	$\dfrac{K\left[c_{i1}-4(c_{p1}+c_{d1})\right]}{1+2c_{p1}}$	$\dfrac{K\left[c_{p1}(2-c_{i1})+2c_{d1}+c_{i1}/2-1\right]}{1+2c_{p1}}$
	1 $c_{p1}=T_0/T$	$-4c_{p1}/(1+2c_{p1})$ $c_{i1}=T/T_i$	$(2c_{p1}-1)/(1+2c_{p1})$ $c_{d1}=T_d/T$

Doplnok – kvalita regulácie frekvenčná oblasť

Closed Loop



Open loop : system H(s) ; system with controller F(s).H(s)

Closed loop : $H_{CL}(s) = F(s).H(s) / [1 + F(s). H(s) G(s)]$

Steady state error : F(s) must contains the internal model of the reference (the transfer function that generates Yref(t) from the Dirac impulse ;
e.g. step = (1/s) * Dirac ; ramp = $(1/s^2)$ * Dirac,...

Closed Loop : Perturbation rejection

Perturbation-output sensitivity function :

$S_{yp}(s) = Y(s) / P(s) = 1 / [1 + F(s).H(s).G(s)]$

Perturbation rejection :

$S_{yp}(0) = 0$ to get a perfect rejection of the perturbation in steady state (controller must contain the classes of perturbation)

and

$|S_{yp}(\omega)| < G \; ; \; \forall \omega$

[Example : $|S_{yp}(\omega)| < 2$ (6dB) ; $\forall \omega$]

If the energy of the perturbation is concentrated in a given frequency band, the $|S_{yp}(\omega)|$ should be limited in this band.

Controller Design

In order to design and tune a controller :

1) To specify the desired control loop performances

   Regulation and tracking : rise time and max overshoot

   or bandwidth and resonance

2) To choose a suitable controller design method

3) To know the dynamic model of the plant to be controlled

   => control model

Control model :

- Non parametric models : e.g. frequency response, step response,…
- Parametric models : e.g. transfer function, differential eq., state eq.

To get the model :

- knowledge type model (based on the physic laws) ; used for plant simulation and design
- identification models (from experimental data)

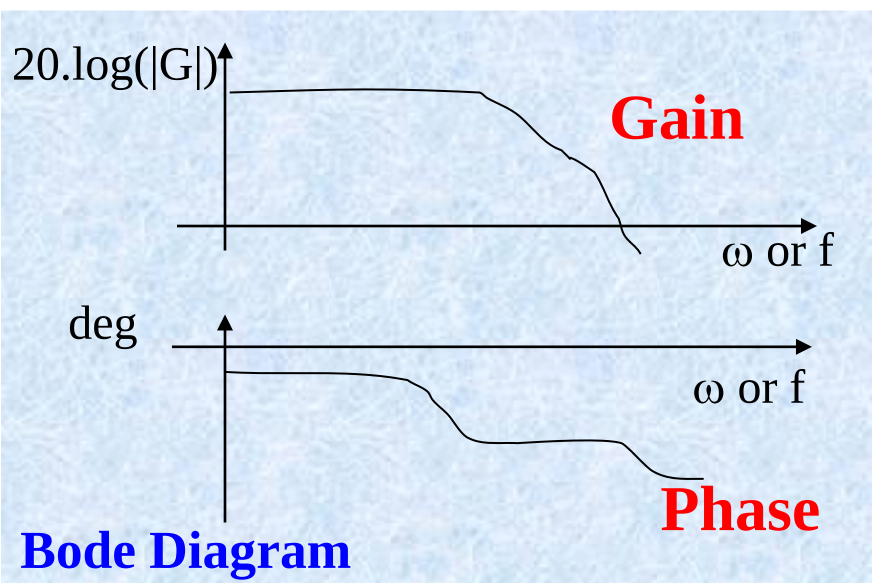
Continuous - time Models : Frequency Domain

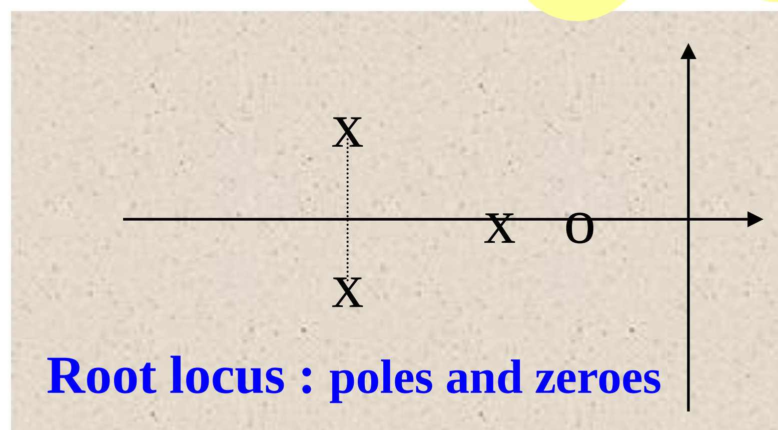
Linear system **H(s) : transfer function**

$u(t) = e^{j\omega t}$ → **System** → $y(t) = G(j\omega) \,.\, e$...

$u(t) = e^{st}$ → **System** → $y(t) = G(s) \,.\, e^{s}$...

Note:
State Equation
Differential Eq.
Transfer function
Observability,
Controlablity

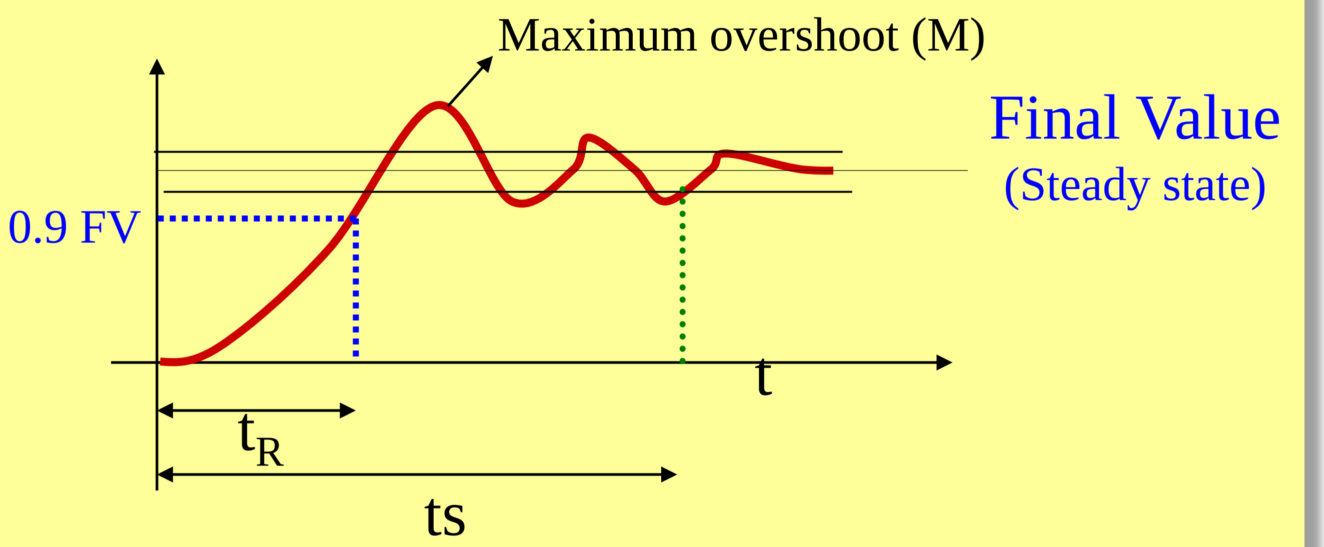




Nyquist, Nichols,...

Continuous - time Models : Time responses

Response of a dynamic system for a step input



Example : 1^{st} Order
$H(s) = G/(1+sT)$
FV = G
$t_R = 2.2\ T$
$t_S = 2.2\ T$ (for 10% FV)
$t_S = 3\ T$ (for 5% FV)
M = 0

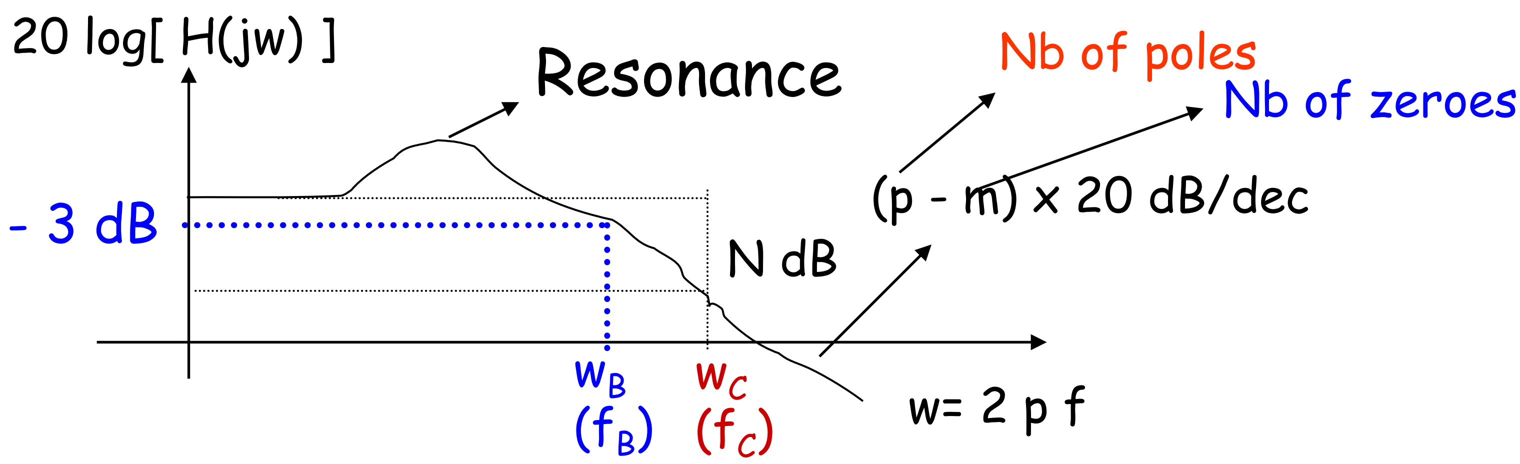
t_R : Rise Time ; define as the time needed to attain 90% of the final value ; or as the time needed for the output to pass from 10 to 90% of the final value

t_S : Settling Time ; define as the time needed for the output to reach and remain within a tolerance zone around the final value (±10%, ± 5%, ±1%,…)

FV : Final Value ; a fixed output value obtained for $t \to \infty$

M : Maximum Overshoot ; expressed as a percentage of the final value

Continuous - time Models : Frequency responses



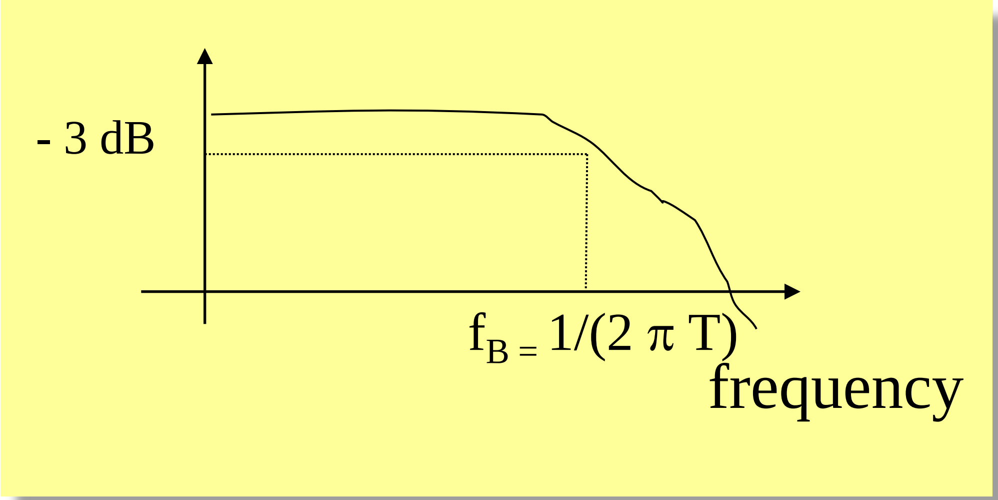
f_B : **Bandwidth** ; the frequency from which the zero-frequency (steady state) gain G(0) is attenuated by more than 3 dB ; $G(w_B) = G(0) - 3dB$ or $G(w_B) = 0.707 \, . \, G(0)$

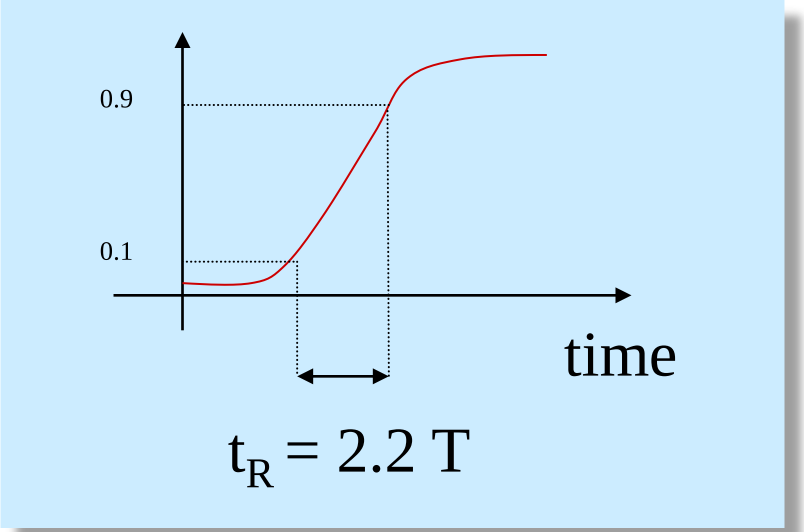
f_C : **Cut-off frequency** ; the frequency from which the attenuation is more than N dB ;

$$G(w_C) = G(0) - NdB$$

Q : Resonance factor ; the ratio between the gain corresponding to the maximum of the frequency response curve and the value G(0)

Reciprocity : Time / Frequency





$$f_B \cong 0.35 / t_R$$

Closed Loop : Margins

Module Margin :

$\Delta M = |1 + H(j\omega)|_{min} = |S^{-1}_{yp}(j\omega)|_{min}$

Measure of perturbation rejection and robustness of non linearity and time variable parameters

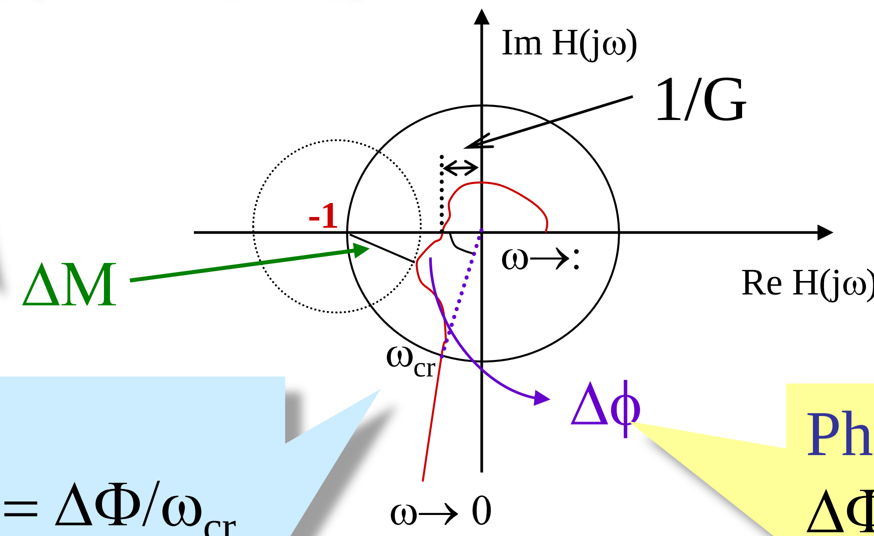
Typical : ΔM /0.5 (-6dB) [min: 0.4 (-8dB)]

Gain Margin :

$DG = 1 / |H(jw_{180})|$

for $F(w_{180}) = -180°$

Typical : G /2 (6dB) [min: 1.6 (4dB)]



Delay Margin :

$\Delta\Phi = \omega_0.\tau$; $\Delta\tau = \Delta\Phi/\omega_{cr}$

additional delay that could be tolerate by the open loop system without instability for the closed loop system

Phase Margin :

$\Delta\Phi = 180° - \Phi(\omega_{cr})$

for $|H(j\omega_{cr}) = 1$

ω_{cr} : crossing pulsation

Typical : 30 ° [ΔΦ [60 °